

Disinflations in a model of imperfectly anchored expectations ^{*}

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Abstract

We study disinflations under imperfect credibility of the central bank. We propose a framework to model imperfectly credible announcements and use it to study the distribution of the output cost for a given disinflation. Imperfect credibility is modeled as the extent to which agents rely on adaptive learning to form expectations. Lower credibility increases the mean, variance, and skewness of the distribution of the sacrifice ratio. When credibility is low, disinflations become very costly for adverse realizations of the shocks. But, an opportunistic disinflation, a disinflation implemented after a period of below trend inflation, can significantly lower the sacrifice ratio. With simulated data, we reinterpret the reduced form evidence in sacrifice ratio regressions. Coefficient estimates from these regressions can be misleading for policymakers considering the cost of disinflation.

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1 Introduction

Advanced economies have succeeded in keeping inflation low after the high inflation rates of the 1970's. Other economies have not. For example, in Argentina, inflation has once again become the main macroeconomic concern. The case for price stability is by now well-established and theoretical studies of disinflations, like [Ascari and Ropele \(2013\)](#) and [Ireland \(1995\)](#), make clear that the long-term welfare gains from low inflation tend to exceed the short-term cost of reducing it. But empirical studies on the cost of disinflation, such as [Cecchetti and Rich \(2001\)](#), [Brito \(2010\)](#), [Brito and Bystedt \(2010\)](#), [Mazumder \(2014\)](#), and [Katayama et al. \(2015\)](#), suggest that considerable uncertainty remains over what determines the short-term cost of disinflation.

In this paper, we extend the study of disinflation in two ways. First, following [Ball \(1994\)](#), we significantly extend the sample, relative to previous studies, on the cost of disinflation.¹ We use this sample to show that the variance of the distribution of the sacrifice ratio is large and that key determinants of the sacrifice ratio, like the size of the disinflation or its duration, explain little of its variation. Second, we propose a method to model imperfectly credible announcements. We use this method to study how the distribution of the sacrifice ratio changes as the degree of credibility changes in the context of an estimated sticky-price structural model. What the distribution makes clear is that the cost of any realized disinflation may be more affected by shocks (luck) than by the design of the disinflation policy.

By expanding the analysis to the determinants of the distribution, we are able to bridge the theoretical literature on the cost of disinflations with the econometric evidence from regression results. Theoretical studies of the cost of disinflation such as [Ascari and Ropele \(2012,b, 2013\)](#) show that medium-scale monetary models of the kind proposed by [Christiano et al. \(2005\)](#) can yield perfect foresight paths that are consistent with the mean

¹The cost of disinflation is measured by the sacrifice ratio which is defined as the cumulative percentage of output growth lost for decreasing the annual inflation rate by one percent.

sacrifice ratio in the data, a number which ranges somewhere between one and four.² But, perfect foresight simulations do not account for the uncertainty surrounding estimates of the mean sacrifice ratio, nor the significant variation of the sacrifice ratio observed in the data, which we find can range from -50 to 37. Using an estimated structural model, we generate artificial disinflations to show that the empirical regressions of the sacrifice ratio may be misleading.

We propose a novel way to model imperfectly credible announcements in a standard sticky-price model. We extend the approach of [Evans and Honkapohja \(2001\)](#) by replacing the expectations operator with expectations that combine forecasts based on the solution for anticipated structural changes of [Kulish and Pagan \(2017\)](#) with forecasts based on standard adaptive learning. The former forecast captures the forward-looking component of the representative agent's beliefs, while adaptive learning captures the backward-looking component. In our model, the weight that agents place on adaptive learning to form expectations represents the extent to which expectations are anchored. This weight can also be thought to govern the degree of credibility because it determines the impact of a policy announcement.

Our notion of credibility, however, is not the rational expectations notion which arises with time-inconsistency and commitment policy.³ A property of rational expectations equilibrium modeling is that beliefs are always consistent with the policy strategy. In our case, data influences the evolution of beliefs, which allows them to be inconsistent with the policy strategy for some time. This notion of credibility speaks to the view of policymakers exemplified by [Kohn \(2009\)](#) on inflation stabilization. He emphasizes that outcomes over the medium term are an important determinant of the extent to which long-run expectations are anchored to policy objectives.⁴

²The range of the mean sacrifice ratio comes from [Ball \(1994\)](#), [Cecchetti and Rich \(2001\)](#), and [Gonçalves and Carvalho \(2009\)](#).

³It is also distinct from the framework of [Cogley et al. \(2015\)](#), where agents' uncertainty and learning refer only to the central bank's policy rule.

⁴See the 2009 speech by Donald Kohn, former Vice Chairman of the Federal Reserve, available [here](#). [Eusepi et al. \(2015\)](#) present evidence from survey data which is consistent with the view of Kohn showing

Using this model of imperfect credibility, we find that a lower degree of credibility not only increases the mean cost of disinflation but also significantly increases the variance and skewness of the distribution of the sacrifice ratio for all disinflation policies. The design of a disinflation policy such as whether it is anticipated or unanticipated, gradual or cold turkey, shapes the distribution. For example, pre-announcing a disinflation shifts the distribution to the left and reduces right skewness even at low levels of credibility.

Our modeling framework also allows us to quantify the gains from an opportunistic approach to disinflation, when a policymaker takes advantage of shocks that lower inflation and inflation expectations to announce and implement a lower inflation target.⁵ There is evidence that many central banks used this type of strategy in the early 1990's when inflation targeting regimes were first established.⁶ An opportunistic approach to disinflations reduces the sacrifice ratio. The gains are large enough that opportunism can trump credibility. For any disinflation policy, a policymaker with low credibility can often achieve a sacrifice ratio behaving opportunistically that is lower than what a policymaker with high credibility can expect on average.

Finally, we use the distribution of the sacrifice ratio implied by the structural model to re-examine the evidence that comes from regressions results. We find that the coefficient estimates are misleading for policymakers contemplating a disinflation policy for two reasons. First, empirical measures of the sacrifice ratio rely on statistical filters to compute trend inflation and the output gap, which biases the estimates. Second, regressions on observable characteristics of disinflations pool episodes from different conditional distributions, which give rise to an omitted variable bias if non-observable determinants like the degree of credibility are not taken into account. What matters for a policymaker wishing to disinflate is the conditional distribution of the sacrifice ratio that is associated

that long-horizon forecasts of inflation are, in fact, correlated with short-term forecast errors.

⁵Bomfim and Rudebusch (2000), Orphanides and Wilcox (2002), and Aksoy et al. (2006) study different aspect of opportunistic disinflations.

⁶For example, the Reserve Bank of Australia formalized its inflation targeting framework after a significant fall in inflation in the early 1990's. Bomfim and Rudebusch (2000) note that opportunistic policies were discussed in Federal Reserve FOMC meetings in the late 1980's.

with her degree of credibility and with the characteristics of the disinflation policy under consideration. The rest is noise.

The rest of the paper is structured as follows. Section 2 extends the sample of disinflation episodes and reproduces standard regressions to establish reduced-form evidence on the determinants of the sacrifice ratio. Section 3 introduces the framework for imperfectly credible announcements. Section 4 applies this framework to illustrate, analytically, the relation between the sacrifice ratio and credibility in a simple model. Section 5 uses an estimated structural model to study the conditional distribution of the sacrifice ratio and to interpret the reduced-form evidence of Section 2. Section 6 concludes and offer a suggestion for future research.

2 Reduced-form evidence on the sacrifice ratio

The empirical literature on the determinants of the sacrifice ratio largely follows [Ball \(1994\)](#), who proposes a way to identify a disinflation episode and calculate its associated sacrifice ratio. Ball regresses the sacrifice ratio on the observable characteristics of the disinflation such as its size and duration and the peak level of inflation at the start of the disinflation to infer how the sacrifice ratio may vary with changes in policy. These standard regressions are often augmented with other controls including measures of credibility ([Gonçalves and Carvalho \(2009\)](#), [Mazumder \(2014\)](#)). But, as [Brito \(2010\)](#) and [Katayama et al. \(2015\)](#) point out, the results of these studies do not appear to be robust. In this section, we follow this methodology to construct an up-to-date sample of the sacrifice ratio to establish reduced-form evidence.

We construct an unbalanced panel data set containing quarterly CPI and real GDP growth from 1960 to 2014 for 42 countries. Our data set is assembled from data obtained from the OECD, the World Bank and the Federal Reserve Economic Database.⁷ As in

⁷We use annual GDP for countries with long histories of quarterly CPI data that did not have corresponding quarterly GDP data. Places where this substitution is made are clearly marked in our

Ball (1994), a disinflation episode is defined as a two-percentage point or greater decrease in trend CPI inflation from peak to trough, where trend inflation is calculated as a nine quarter moving average. Peaks and troughs are defined as points where trend inflation is higher or lower than the previous and following four quarters, respectively. Using this method, we identify 150 disinflation episodes, a larger sample than the 25 episodes of Ball (1994) or the 58 episodes of Gonçalves and Carvalho (2009).⁸

The sacrifice ratio, SR , of each disinflation episode is defined as

$$SR = -\frac{1}{\Delta\pi} \sum_{t=Peak}^{Trough} \left(\frac{Y_t - \bar{Y}_t}{\bar{Y}_t} \right), \quad (1)$$

where the denominator, $\Delta\pi$, is the change in trend inflation. The numerator is the sum of output losses, measured as the deviation between log output, Y_t , and trend log output, \bar{Y}_t . Trend log output is approximated by the line that connects log output in the quarter that the disinflation starts to the level of log output four quarters after the trough. Tables A1 and A2 in the appendix report the dates, duration, size, and sacrifice ratio estimates for our sample.

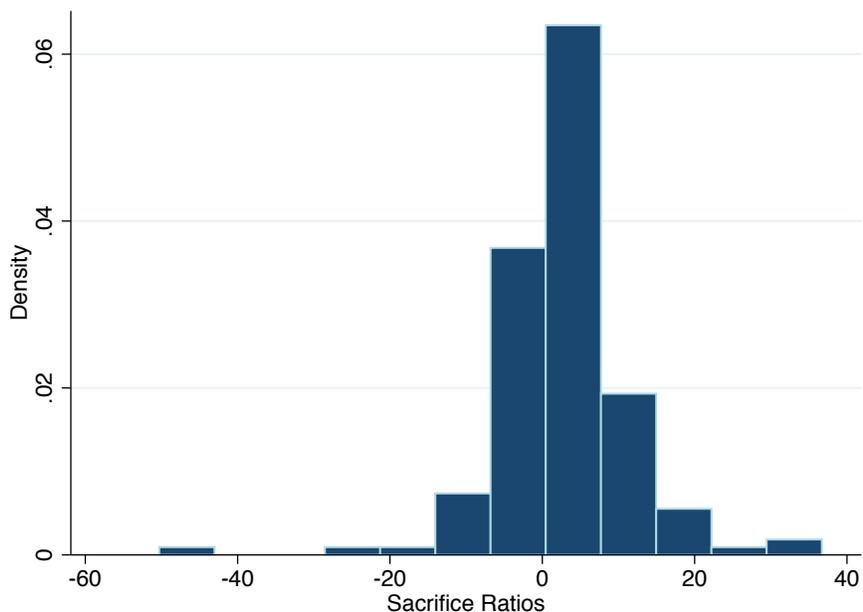
Figure 1 shows the distribution of the sacrifice ratio in our sample. The mean sacrifice ratio is 3.08, which accords well with the existing literature. For instance, Gonçalves and Carvalho (2009) find a mean sacrifice ratio of 5.6, Ball (1994) finds 1.4, and for the Volcker disinflation Blinder (1987) finds 4.2, and Sachs (1985) 2.9. The distribution has a standard deviation of 9.04 with sacrifice ratios ranging from -50 to 37.

A common research question in this literature is to assess the cost of a gradual versus a cold turkey disinflation. The standard specification in this case regresses the sacrifice ratio on three observable characteristics of a disinflation episode: the size of the disinflation, the peak level of trend inflation prior to the disinflation, and the duration. The size and

tables. With annual GDP data, we assume GDP grew equally in each quarter. Our dataset is available upon request.

⁸We exclude disinflation episodes for which trend inflation exceeds 50%. Ball (1994) and Gonçalves and Carvalho (2009) do not consider disinflation episodes for which trend inflation exceeds 20%. This restriction, however, omits intended disinflation episodes such as Chile’s disinflation in the 1990’s.

Figure 1: Empirical distribution of sacrifice ratios



duration proxy for gradualism, while the level of trend inflation proxies for the degree of nominal rigidities. The results of such a regression are given in Table 1.

The coefficient estimates are in line with those in the literature. The size of the disinflation and the peak of trend inflation are negatively correlated with the sacrifice ratio, while the duration of the disinflation is positively correlated, which implies that gradualism increases costs. The statistical significance and sign of these coefficients, however, are not robust across different specifications. In addition, the regressors explain only a small amount of the overall variation of the sacrifice ratio ($R\text{-squared} = 0.129$).

The disinflation literature has identified the degree of credibility as an important determinant of the sacrifice ratio. To the extent that this is true and that disinflations in the sample come from economies with varying degrees of credibility, the regressions in Table 1 would suffer from an omitted variable bias. In practice, it is not easy to find good measures of credibility, but we propose to use data from [Dreher et al. \(2008\)](#) and [Dreher et al. \(2010\)](#) on central bank governance to construct a measure of credibility by looking at disinflations that occur the year of, or the year after, a new central bank governor is

Table 1: Determinants of the sacrifice ratio

| VARIABLES | (1) SR | (2) SR | (3) SR | (4) SR |
|--|-------------------|--------------------|-----------------|-----------------|
| Change Trend Inflation ($\Delta\pi$) | -0.47** (0.20) | | | -0.32 (0.21) |
| Peak Trend Inflation | | -0.36*** (0.11) | | -0.22 (0.14) |
| Duration | | | -0.02 (0.21) | 0.09 (0.22) |
| Observations | 150 | 150 | 150 | 150 |
| R-squared | 0.09 | 0.11 | 0.00 | 0.13 |

Notes: Estimated standard errors are reported in parentheses and clustered at the country level. *** significant at 1%; ** significant at 5%; * significant at 10%.

appointed.⁹ An advantage of this measure, compared to others, is that it goes back to 1975, which covers the majority of our sample. In addition, we consider country fixed effects to control for any other fixed difference between countries that do not vary over time.

Table 2 shows the regression results using our measure of credibility. We find that the appointment of a new central bank governor prior to a disinflation lowers the average sacrifice ratio. It also changes the relationship of the sacrifice ratio with the three observable determinants. In each case, credibility weakens the relationship further suggesting that there is no correlation with these three determinants when the disinflation is credible. Although this measure of credibility is imprecise, the results are consistent with other measures of credibility considered in the literature.¹⁰

What these regressions reveal is that there is substantial unexplained variation in the sacrifice ratio. The strongest relationship observed in the regression is between the size of the disinflation and the sacrifice ratio. This relationship suggests that a central bank should disinflate by as much as possible to minimize costs (see Table 1). However, when credibility is considered the trade-off weakens, which suggests that there is a nonlinear relationship between the observed determinants of the sacrifice ratio and credibility (see

⁹The appointment of a new central bank governor is arguably a signal of a change in policy.

¹⁰See estimates in [Gonçalves and Carvalho \(2009\)](#), [Brito \(2010\)](#) and [Mazumder \(2014\)](#).

Table 2: Credibility and the sacrifice ratio

| VARIABLES | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
|--|--------|--------|----------|--------|--------|--------|---------|--------|
| | SR | SR | SR | SR | SR | SR | SR | SR |
| Change Trend Inflation ($\Delta\pi$) | -0.63* | -0.49 | | | | | -0.26 | -0.35 |
| | (0.37) | (0.42) | | | | | (0.24) | (0.33) |
| Change \times New Appointment | 0.27 | 0.33 | | | | | -0.18 | 0.31 |
| | (0.44) | (0.54) | | | | | (0.35) | (0.40) |
| Peak Trend Inflation | | | -0.52*** | -0.32* | | | -0.42** | -0.15 |
| | | | (0.19) | (0.17) | | | (0.19) | (0.16) |
| Peak \times New Appointment | | | 0.33 | 0.20 | | | 0.41** | 0.07 |
| | | | (0.21) | (0.19) | | | (0.20) | (0.16) |
| Duration | | | | | -0.08 | -0.14 | 0.04 | -0.02 |
| | | | | | (0.31) | (0.31) | (0.30) | (0.31) |
| Duration \times New Appointment | | | | | 0.07 | 0.01 | 0.09 | -0.05 |
| | | | | | (0.41) | (0.41) | (0.41) | (0.42) |
| New Appointment | -1.86 | -2.78 | -4.05 | -2.62 | -1.60 | -1.10 | -5.22 | -2.89 |
| | (3.05) | (4.33) | (2.94) | (2.95) | (6.08) | (6.71) | (7.47) | (7.97) |
| Observations | 126 | 126 | 126 | 126 | 126 | 126 | 126 | 126 |
| R-squared | 0.11 | 0.07 | 0.15 | 0.05 | 0.00 | 0.03 | 0.17 | 0.08 |
| Country Fixed Effects | No | Yes | No | Yes | No | Yes | No | Yes |

Notes: We only have data on central bank governance dating back to 1975, which is why the number of observations decreases in these regressions. Estimated standard errors are reported in parentheses and clustered at the country level. *** significant at 1%; ** significant at 5%; * significant at 10%.

Table 2).

The framework we propose to study disinflations confirms some, though, not all of these reduced form findings. In particular, it confirms that the determinants of the sacrifice ratio considered in these regressions should be expected to explain little variation given empirically plausible shocks and that the degree of credibility plays a significant nonlinear role in shaping the distribution of the sacrifice ratio. In addition, using simulated data, it becomes clear that the statistical filters used in the empirical literature to identify disinflations and compute the sacrifice ratio distort the true trade-offs that policymakers face.

3 Modeling imperfectly credible announcements

We propose a general framework to model imperfectly credible announcements.¹¹ A disinflation policy in standard models is a permanent change in the inflation target,

¹¹Additional details can be found in the online appendix.

which is one of the structural parameters. In general, we assume that prior to the change in policy the economy follows the n linearized equations of the form

$$y_t = \mathbf{\Gamma} + \mathbf{A}y_{t-1} + \mathbf{B}\hat{\mathbb{E}}_t y_{t+1} + \mathbf{D}\varepsilon_t, \quad (2)$$

where y_t is a $n \times 1$ vector of state and jump variables and ε_t is a $l \times 1$ vector of exogenous variables.¹² When the new policy is implemented at T^* , the structural equations change to

$$y_t = \mathbf{\Gamma}^* + \mathbf{A}^*y_{t-1} + \mathbf{B}^*\hat{\mathbb{E}}_t y_{t+1} + \mathbf{D}^*\varepsilon_t \quad (3)$$

which holds for all $t \geq T^*$. In the context of a disinflation, Equation (2) would correspond to the approximation around the high inflation steady state, π^H , and Equation (3) would correspond to the approximation around the low inflation steady state, π^L .

In the case of a policy change, an important question is how one models the evolution of beliefs. We follow [Gibbs \(2017\)](#) in assuming that the representative agent combines forecasts from different models to form expectations. In particular, we assume $\hat{\mathbb{E}}_t y_{t+1}$ is a linear combination of the form

$$\hat{\mathbb{E}}_t y_{t+1} = \lambda \mathbb{E}_t^S y_{t+1} + (1 - \lambda) \mathbb{E}_t^R y_{t+1} \quad (4)$$

where $\mathbb{E}_t^S y_{t+1}$ is a structural forecast, $\mathbb{E}_t^R y_{t+1}$ is a reduced form forecast, and $0 \leq \lambda \leq 1$. The structural forecast is computed independently of λ using the time-varying VAR solution for structural change under rational expectations of [Kulish and Pagan \(2017\)](#), which assumes the representative agent knows Equations (2) and (3) when constructing this forecast. Thus, when $\lambda = 1$, $\mathbb{E}_t^S y_{t+1}$ is a rational expectations forecast. Otherwise, $\mathbb{E}_t^S y_{t+1}$ anchors beliefs to what would be expected to happen if the policy were

¹²Without loss of generality, we take ε_t to be white noise. All matrices in Equation (2) conform to the specified dimensions. The specification may be further generalized as in [Binder and Pesaran \(1995\)](#) to allow additional lags of y_t as well as expectations at different horizons and from earlier dates.

fully credible. The reduced form forecast $\mathbb{E}_t^R y_{t+1}$ on the other hand is computed under adaptive learning following [Evans and Honkapohja \(2001\)](#) by assuming that agents know the VAR functional form of the solution, $y_t = \mathbf{C} + \mathbf{Q}y_{t-1} + \mathbf{G}\varepsilon_t$. Time t estimates of the coefficient in \mathbf{C} and \mathbf{Q} are obtained using past data via a standard constant gain recursive least squares algorithm. These we denote by $\tilde{\mathbf{C}}_t$ and $\tilde{\mathbf{Q}}_t$, respectively. When $\lambda = 0$, the solution coincides with the standard adaptive learning solution of [Evans and Honkapohja \(2001\)](#). Thus, Equation (4) nests rational expectations and adaptive learning as special cases. Outside these extremes, we refer to Equation (4) as *imperfectly anchored expectations*. This is because the parameter λ measures the extent to which expectations are anchored, i.e. the degree to which the forecasting function, $\hat{\mathbb{E}}_t y_{t+1}$, is invariant to incoming data.¹³

As derived in the online appendix, substituting Equation (4) into Equation (2) yields a time-varying VAR solution of the form

$$y_t = \hat{\mathbf{C}}_t + \hat{\mathbf{Q}}_t y_{t-1} + \hat{\mathbf{G}}_t \varepsilon_t, \quad (5)$$

where

$$\begin{aligned} \hat{\mathbf{C}}_t &= [\mathbf{I} - \mathbf{B}(\lambda \mathbf{Q}_{t+1} + (1 - \lambda) \tilde{\mathbf{Q}}_t)]^{-1} (\mathbf{\Gamma} + \lambda \mathbf{B} \mathbf{C}_{t+1} + (1 - \lambda) \mathbf{B} \tilde{\mathbf{C}}_t) \\ \hat{\mathbf{Q}}_t &= [\mathbf{I} - \mathbf{B}(\lambda \mathbf{Q}_{t+1} + (1 - \lambda) \tilde{\mathbf{Q}}_t)]^{-1} \mathbf{A} \\ \hat{\mathbf{G}}_t &= [\mathbf{I} - \mathbf{B}(\lambda \mathbf{Q}_{t+1} + (1 - \lambda) \tilde{\mathbf{Q}}_t)]^{-1} \mathbf{D} \end{aligned}$$

that describes the evolution of the economy prior to the implementation of the new policy. The solution includes both backward ($\tilde{\mathbf{Q}}_t$) and forward-looking (\mathbf{Q}_{t+1}) information.

¹³The assumption that the structural forecast is consistent with the rational expectations solution of the model when $\lambda = 1$ is not a required assumption. One could consider a more general formulation in which the structural forecast is not model-consistent at $\lambda = 1$ so that even when the policy is perfectly credible, forecasts would not coincide with the standard rational expectations solution as in our case. Such a formulation could be used to allow for the central bank and the private sector to have different beliefs as is the case in [Honkapohja and Mitra \(2005\)](#) or [Preston \(2008\)](#). However, our formulation in Equation (4) does not distinguish between private sector and policymakers' expectations.

The backward-looking information comes from the estimated coefficients from adaptive learning expectations and the forward-looking information comes from the announcement regarding future policy changes. In this context, λ serves as a natural measure of credibility because it governs the impact of an announcement. When $\lambda = 0$, an announcement would have no impact on the reduced-form and forecasting functions would respond only to past data. As λ increases, forecasting functions respond more strongly to an announcement.

One might think that in practice λ may change over time as a function of the success of policymakers in achieving known objectives. Because a policymaker's credibility is set at the time she implements a disinflation policy, and because we are interested in how a given level of credibility affects a single disinflation outcome, we do not attempt to formalize the process by which credibility may evolve over time.¹⁴

Our setup, when applied to disinflations, may seem similar to other well-known strategies to model imperfect credibility in the literature. For example, [Ball \(1995\)](#), [Goodfriend and King \(2005\)](#), and [Ascari and Ropele \(2013\)](#) all model inflation expectations in some form as $\hat{\mathbb{E}}_t \pi_{t+1} = \lambda \mathbb{E}_t \pi_{t+1} + (1 - \lambda) \bar{\pi}^H$, where $\bar{\pi}^H$ is the old inflation target, $\mathbb{E}_t \pi_{t+1}$ is the forward-looking expectation of inflation, and λ , which can be interpreted as the probability that the policymaker reverts to the old target, proxies for a measure of credibility. Our notion of credibility differs in two important ways. First, in their most basic formulation, credibility applies only to inflation expectations and the probability of reverting to the old target is taken to be constant. An implication of these assumptions is that beliefs would never adjust to be consistent with the new inflation target. In contrast, in our case beliefs will ultimately be consistent with the new inflation target because of adaptive learning.¹⁵

¹⁴This is an interesting question for future research. Endogenizing this process along lines considered by [Marcet and Nicolini \(2003\)](#) and [Sargent et al. \(2009\)](#) for the gain parameter in adaptive learning is one way to implement this assumption.

¹⁵The online appendix discusses the E-stability properties of imperfectly anchored expectations. In general, the conditions for stability are loosened relative to the standard adaptive learning assumption and depend on the value of λ .

Second, adaptive learning makes expectations more responsive to shocks. This is important for assessing the role of luck in disinflation outcomes since in practice many disinflations appear to have been implemented following unintended falls in trend inflation. Take for instance the extreme case of $\lambda = 0$. Good luck shocks in our setup can still lower $\mathbb{E}^R \pi_{t+1}$ through the reduced form forecast but inflation expectations would remain fixed at $\bar{\pi}^H$ by construction in the alternative specification. So even with no credibility, beliefs can favorably evolve with data in our case. This difference regarding the role of shocks would persist even if λ were exogenously time-varying as in [Ascari and Ropele \(2013\)](#), which we discuss in detail in the online appendix.¹⁶

Imperfect credibility can give rise to a sacrifice ratio because it makes a fraction of expectations backward looking. Other backward looking assumptions like habit formation and price and wage indexation that have been considered in the disinflation literature can also give rise to a sacrifice ratio. In this sense, these assumptions can substitute for imperfect credibility. But our setup can be applied to any model that incorporates these assumptions and so in principle can nest such specifications as special cases. The main advantage of our framework is that in addition to nesting these alternatives it allows us to consider imperfectly credible announcements.

Lastly, combining expectations as we propose provides a parsimonious way to introduce anticipated state-contingent policy changes into the traditional adaptive learning framework. [Evans et al. \(2009\)](#) and [Mitra et al. \(2013\)](#) have also proposed ways to incorporate the anticipated path of a policy variable into an adaptive learning framework. They propose giving agents perfect foresight of the path of a policy instrument into the infinite future while assuming that agents form adaptive forecasts of all other variables.

¹⁶[Huh and Lansing \(2000\)](#) also study disinflation under imperfect credibility with adaptive learning. However, they do not consider the distribution of sacrifice ratios or anticipated policies. [Hommes and Lustenhouwer \(2015\)](#) consider the case of heterogeneous expectations where a continuum of agents select between a fundamentals based forecasting rule and naive forecasting rule. The degree to which agents rely on the fundamentals based rule is interpreted as a measure credibility, which corresponds closely with our notion of credibility. Our model is also different to that of [Erceg and Levin \(2003\)](#) study of inflation persistence under imperfect credibility. They model imperfect credibility as a filtering problem where agents must disentangle transitory from persistent changes of the policy rule.

Likewise, [Eusepi \(2005\)](#), [Preston \(2006\)](#), and [Eusepi and Preston \(2010\)](#) have also studied state contingent policy changes under learning, but consider only the case when the changes are unanticipated. These papers use the infinite horizon learning setup introduced by [Preston \(2005\)](#), while we use the standard Euler equation learning framework. One advantage of our choice is that it allows for imperfect credibility to be added to the large class of linearized models already in use.¹⁷

4 Analytical results on the sacrifice ratio

In this section, we illustrate how imperfectly anchored expectations determines the sacrifice ratio in a New Keynesian model simple enough to allow for analytical results. The model is the standard three-equation IS-curve, Phillips curve, and a monetary policy rule:

$$\begin{aligned}x_t &= \hat{\mathbb{E}}_t x_{t+1} - (r_t - \hat{\mathbb{E}}_t \pi_{t+1}) \\ \pi_t &= \beta \hat{\mathbb{E}}_t \pi_{t+1} + \psi x_t \\ r_t &= \bar{\pi} + \alpha (\pi_t - \bar{\pi}).\end{aligned}$$

where x_t is the output gap, π_t is inflation, r_t is the nominal interest rate, and $\bar{\pi}$ is the inflation target. β is set to one so that in steady state $\pi_t = \bar{\pi}$, $r_t = \bar{\pi}$ and $x_t = 0$ and the Taylor principle is satisfied, i.e. $\alpha > 1$.

A disinflation policy corresponds to a permanent decline of the inflation target, $\bar{\pi}$, and its output cost depends on the expected path of inflation. To see this, substitute the Phillips curve and monetary policy rule into the IS-curve and iterate forward to obtain $x_t = (\alpha - 1) \sum_{j=1}^{\infty} (1 + \alpha\psi)^{-j} (\bar{\pi} - \hat{\mathbb{E}}_t \pi_{t+j})$, where we have used the fact that in a stable equilibrium $\lim_{j \rightarrow \infty} \hat{\mathbb{E}}_t x_{t+j} = 0$. This expression shows that the cost of a disinflation depends on the behavior of inflation expectations relative to the inflation target. To the

¹⁷Extending the analysis to the setup of [Preston \(2005\)](#) is a worthwhile avenue for future research.

extent that inflation expectations do not fall as much as the inflation target does, that is $(\bar{\pi} - \hat{\mathbb{E}}_t \pi_{t+j}) < 0$, there is an output cost.

We first consider the initial impact of an unanticipated disinflation under imperfectly anchored expectations.¹⁸ The central bank lowers the inflation target from π^H to π^L . The inflation target is π^H until some time period T^* when the central bank announces and implements π^L . The output gap and inflation in period T^* can be shown to be given by the expressions below

$$x_{T^*} = (1 - \lambda) \frac{(\pi^L - \pi^H)(\alpha - 1)}{(1 + \alpha\psi)}$$

$$\pi_{T^*} = (1 - \lambda) \frac{(1 + \psi)}{(1 + \alpha\psi)} \pi^H + \frac{(\alpha - 1)\psi + \lambda(1 + \psi)}{(1 + \alpha\psi)} \pi^L.$$

When expectations are perfectly anchored, the policy is perfectly credible ($\lambda = 1$) and the disinflation is achieved at the time of implementation without any loss of output. However, with imperfect credibility ($\lambda < 1$), a disinflation gives rise to an output loss because expectations do not immediately adjust to the new lower inflation target. The central bank, therefore, must contract demand to move inflation and inflation expectations to the desired level.

In the case of an anticipated disinflation, the central bank announces the future implementation of the new target. For simplicity, let the policy be announced a period in advance, that is $T^a = T^* - 1$, where T^a refers to the time of the announcement, and T^* now stands for the time of implementation. Using the [Kulish and Pagan \(2017\)](#) solution, one can show that the output gap and inflation at the time of the announcement, T^a ,

¹⁸We refer the interested reader to the online appendix for a derivation of the expressions shown here, for an analysis of the convergence properties towards the lower inflation regime, and for a comparison with the expectational specification of [Ascari and Ropele \(2013\)](#).

are given by the expressions below:

$$x_{T^a} = \lambda \frac{(\pi^H - \pi^L)(\alpha - 1)}{1 + \alpha\psi}$$

$$\pi_{T^a} = \pi^H - \lambda \frac{(\pi^H - \pi^L)(1 + \psi)}{1 + \alpha\psi}$$

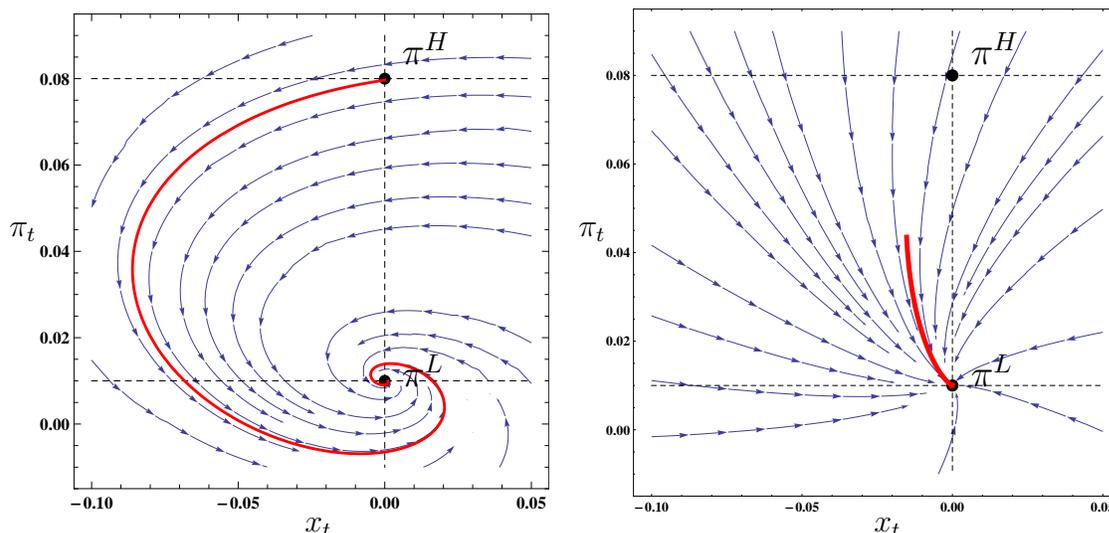
These expressions show that the extent to which expectations are imperfectly anchored, λ , serves as a natural measure of credibility. This is because the impact of an announcement, the anticipation effect, depends on λ . In the extreme case in which $\lambda = 0$, the pre-announcement has no impact, output does not change, and inflation remains at the old higher target, π^H . However, if $\lambda > 0$, the announcement influences output and inflation. In particular, an anticipated disinflation results in a boom ($x_t > 0$). This is because the news of an impending disinflation causes inflation expectations and actual inflation to fall. As a result, inflation falls below the current inflation target, π^H , so the central bank reduces the nominal interest rate. The decrease in the nominal interest rate causes a decrease in the real interest rate and a corresponding boom in output.

Provided $\lambda < 1$, inflation expectations and inflation would not be at the new inflation target once the policy is implemented. Convergence to the new steady state then hinges on adaptive learning. The adaptive learning rule in this simple case is a recursively estimated mean:

$$\mathbf{C}_t = \mathbf{C}_{t-1} + \gamma(y_{t-1} - \mathbf{C}_{t-1}). \quad (6)$$

Figure 2 shows the global convergence properties for an unanticipated disinflation. The two large dots on the figure correspond to $(0, \pi^H)$, the high inflation steady state, and to $(0, \pi^L)$, the low inflation steady state. The solid line shows the paths that output and inflation take when the initial condition is the high inflation steady state. We compute paths for $\lambda = 0$ (left) and for $\lambda = 1/2$ (right). With no credibility, inflation can only be lowered through a demand contraction. The path to the new steady state is long and

Figure 2: Global convergence



Notes: Stream plots illustrating convergence to new steady state with $\lambda = 0$ (left) and $\lambda = 0.5$ (right). The solid line indicates the non-stochastic path assuming inflation and output gap are at steady state upon implementation of the new policy.

overshoots once in the neighborhood of the new target. With some credibility, though, inflation and output jump upon the simultaneous announcement and implementation of the policy. In addition, the path to the new steady state is more direct due to an increased anchoring of beliefs. In the case of full credibility, there are no convergence dynamics; beliefs jump from the high steady state to the low steady state without output moving.

The arrows in the graph also provide insight to how shocks would affect the sacrifice ratio. Shocks that push inflation towards the new target put beliefs on a path that is unambiguously less costly.

5 Quantitative Results

In this section, we consider a version of the New Keynesian model of Ireland (2004) to study the distribution of the sacrifice ratio under imperfectly anchored expectations for different types of disinflation policies. We discipline this exercise by exploring the model at its estimated parameter values using recent data. We are not aiming to characterize

the distribution of the sacrifice ratio for any particular country or time period. Instead, we are interested in how imperfectly anchored expectations shapes the distribution of the sacrifice ratio as we vary the policy and degree of credibility for empirically plausible distributions of the shocks. We estimate the model on data from United States (1984Q1 - 2008Q1) and Argentina (2003Q3 - 2014Q1).¹⁹ We use the United States parameter values to study disinflation policies with small shocks and use the parameter values implied by Argentinean data to study disinflation policies with large shocks.

The linearized approximation of the model is given by the equations below.

$$x_t = (z - \ln \beta) - (r_t - \hat{\mathbf{E}}_t \pi_{t+1}) + \hat{\mathbf{E}}_t x_{t+1} + (1 - \omega)(1 - \rho_a) a_t \quad (7)$$

$$\pi_t = (1 - \beta) \bar{\pi} + \beta \hat{\mathbf{E}}_t \pi_{t+1} + \psi x_t - e_t \quad (8)$$

$$r_t = r + \rho_r (r_{t-1} - r) + \rho_\pi (\pi_t - \bar{\pi}_t) + \rho_g (g_t - z) + \rho_x x_t + \varepsilon_{r,t} \quad (9)$$

$$\bar{\pi}_t = (1 - \rho_\pi^*) \bar{\pi} + \rho_\pi^* \bar{\pi}_{t-1} + \mu_a \varepsilon_{a,t} + \mu_e \varepsilon_{e,t} + \mu_v \varepsilon_{v,t} + \mu_z \varepsilon_{z,t} + \mu_\pi \varepsilon_{\pi,t} \quad (10)$$

$$\omega_t = m_t - m_{t-1} - \pi_t + z_t \quad (11)$$

$$m_t = \hat{y}_t + a_t - \frac{1}{g\bar{\pi}\beta^{-1} - 1} (r_t - r) + v_t \quad (12)$$

$$x_t = \hat{y}_t - \omega a_t \quad (13)$$

$$g_t = \hat{y}_t - \hat{y}_{t-1} + z_t \quad (14)$$

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t} \quad (15)$$

$$e_t = \rho_e e_{t-1} + \varepsilon_{e,t} \quad (16)$$

$$v_t = \rho_v v_{t-1} + \varepsilon_{v,t} \quad (17)$$

$$z_t = (1 - \rho_z) z + \rho_z z_{t-1} + \varepsilon_{z,t}, \quad (18)$$

where x_t is the output gap, defined as the deviation of output from a socially efficient level of output; π_t is the gross rate of inflation, that is $\ln(p_t/p_{t-1})$ and $\bar{\pi}_t$ is time-varying target;

¹⁹We use Billion Price Project data in place of official inflation statistics for Argentina.

r_t is the log of the gross nominal interest rate; g_t is the growth rate of output; \hat{y}_t is the percentage deviation from steady state of the log of the stochastically detrended level of output, Y_t/Z_t ; Z_t is labor augmenting productivity, whose growth rate, $z_t = \ln(Z_t/Z_{t-1})$ follows the autoregressive process in Equation (18); m_t is the percentage deviation of stochastically detrended real money balances and ω_t is the percentage deviation of nominal money growth from steady state; a_t is a demand shock, e_t is a cost-push shock, and v_t is a money demand shock. The ε_i 's are identically and independently distributed shocks with zero mean and standard deviation σ_i 's respectively. In steady state, $\pi_t = \bar{\pi}$, $r_t = r$, $g_t = g$ and $r = \bar{\pi} + g - \ln \beta$. We assume that there is a short-run inflation target, $\bar{\pi}_t$, that is time-varying around the long-run inflation target, $\bar{\pi}$, and that it accommodates shocks as shown in Equation (10). A specification like this is required when taking the model to the recent Argentinean data where inflation has been trending upwards for a period of time. But, this specification is also useful because it provides a parsimonious way to model gradual disinflations. For a given change in the long-run inflation target, $\Delta\bar{\pi}$, the persistence parameter ρ_π^* governs the speed with which the new unconditional mean of inflation is implemented.²⁰ All disinflations we consider refer to changes of the *long-run* inflation target, $\bar{\pi}$.²¹

The model is estimated using Bayesian methods as is standard in the literature. Details of the data and estimation are given in the appendix below. We estimate the model under rational expectations and use the estimated parameter values for the disinflation simulation exercises. Alternatively, one could think of estimating the model jointly with λ . However, we do not do this for two reasons. First, as stated above, we are interested in

²⁰ ρ_π^* set to zero corresponds to a cold turkey disinflation as the inflation target falls immediately to its new intended value.

²¹We use a Rotemberg (1982) type specification and assume that the cost of price adjustment is relative to the inflation target. As such, the slope of the Phillips curve does not depend on the level of trend inflation. Under different assumptions the slope of the Phillips curve can depend on the level of trend as in Ascari and Ropele (2009). We leave for future research studying the distribution of the sacrifice ratio under these alternative assumptions. In addition, Ascari and Ropele (2013) show that other nonlinearities could be important in disinflation episodes. The nonlinearity in this paper is in expectations.

Table 3: Estimation results

| Argentinean Data | | | | | | US Data | | | | | |
|------------------------------|------------|---------|--------|------------|---------------|------------------------------|------------|---------|--------|------------|---------------|
| Parameter | Prior Mean | Mode | s.d. | Prior | Prior St. Dev | Parameter | Prior Mean | Mode | s.d. | Prior | Prior St. Dev |
| ρ_a | 0.85 | 0.9179 | 0.0872 | Beta | 0.1 | ρ_a | 0.85 | 0.8592 | 0.0872 | Beta | 0.01 |
| ρ_e | 0.85 | 0.9606 | 0.0345 | Beta | 0.1 | ρ_e | 0.85 | 0.8620 | 0.0345 | Beta | 0.01 |
| ρ_z | 0.5 | 0.3755 | 0.2183 | Beta | 0.2 | ρ_z | 0.5 | 0.0715 | 0.2183 | Beta | 0.2 |
| ρ_v | 0.85 | 0.9178 | 0.0628 | Beta | 0.1 | ρ_v | 0.97 | 0.9632 | 0.0628 | Beta | 0.01 |
| ρ_r | 0.7 | 0.9674 | 0.0587 | Beta | 0.2 | ρ_r | 0.97 | 0.9736 | 0.0587 | Beta | 0.01 |
| ρ_π | 0.8 | 0.3749 | 0.0797 | Normal | 0.5 | ρ_π | 0.9 | 0.7901 | 0.0797 | Normal | 0.1 |
| ρ_g | 0.2 | 0.3228 | 0.0558 | Normal | 0.1 | ρ_g | 0.8 | 0.9662 | 0.0558 | Normal | 0.1 |
| ρ_x | 0.1 | 0.0075 | 0.0062 | Normal | 0.05 | ρ_x | 0.2 | 0.1846 | 0.0062 | Normal | 0.05 |
| ρ_π^* | 0.7 | 0.8044 | 0.0567 | Beta | 0.1 | ρ_π^* | 0.2 | 0.7299 | 0.0567 | Beta | 0.2 |
| π | 0.036 | 0.0454 | 0.0052 | Normal | 0.01 | π | 0.01 | 0.0081 | 0.0052 | Normal | 0.1 |
| μ_a | -0.1 | -0.1669 | 0.1642 | Normal | 0.2 | μ_a | -0.3 | -0.2999 | 0.1642 | Normal | 0.01 |
| μ_e | -0.1 | -0.0597 | 0.1969 | Normal | 0.2 | μ_e | 0.6 | 0.8573 | 0.1969 | Normal | 0.2 |
| μ_z | 0.1 | 0.1603 | 0.1918 | Normal | 0.2 | μ_z | 0.1 | -0.0269 | 0.1918 | Normal | 0.2 |
| μ_v | 0.0 | 0.1776 | 0.0575 | Normal | 0.2 | μ_v | 0.0 | 0.0372 | 0.0575 | Normal | 0.2 |
| Standard deviation of shocks | | | | | | Standard deviation of shocks | | | | | |
| σ_a | 0.02 | 0.0089 | 0.0034 | Inv. Gamma | ∞ | σ_a | 0.015 | 0.0089 | 0.0034 | Inv. Gamma | ∞ |
| σ_e | 0.01 | 0.0061 | 0.0008 | Inv. Gamma | ∞ | σ_e | 0.001 | 0.0139 | 0.0008 | Inv. Gamma | ∞ |
| σ_z | 0.012 | 0.0051 | 0.0018 | Inv. Gamma | ∞ | σ_z | 0.020 | 0.0008 | 0.0018 | Inv. Gamma | ∞ |
| σ_r | 0.02 | 0.0051 | 0.0009 | Inv. Gamma | ∞ | σ_r | 0.010 | 0.0090 | 0.0009 | Inv. Gamma | ∞ |
| σ_π^* | 0.01 | 0.0036 | 0.001 | Inv. Gamma | ∞ | σ_π^* | 0.010 | 0.0004 | 0.001 | Inv. Gamma | ∞ |
| σ_v | 0.07 | 0.0354 | 0.0129 | Inv. Gamma | ∞ | σ_v | 0.070 | 0.0799 | 0.0129 | Inv. Gamma | ∞ |

Notes: Parameter values used for the quantitative exercises. Values were obtained by estimating the model on US and Argentinean aggregate data. Details of the estimation are given in the appendix. The parameter ρ_π^* is set to zero for cold turkey disinflations.

the distribution of the sacrifice ratio as a function of λ , which we vary in our simulations. Second, once learning converges to rational expectations, the solutions are observational equivalent and λ is not likely to be identified in a sample without a disinflation. In addition, inference on structural parameters often does not change when comparing estimated DSGE models under adaptive learning to those estimated under rational expectations. For example, [Slobodyan and Wouters \(2012\)](#) find small differences in the estimates of the key parameters and shocks of the Smets and Wouters model, while [Cole and Milani \(2014\)](#) find similar results in a small scale DSGE model even when they consider multiple different ways to specify agents' beliefs.

The estimated parameter values and shock variances are in [Table 3](#). There are two points to note. The first is that the variances of the shocks are larger for Argentina. The standard deviation of inflation in Argentina is more than four times that of the United States. The second point is that monetary policy is more aggressive in the US. The response to inflation is twice as large in the US. This implies, as we have seen in the analytical section, different output costs for similar sized disinflations.

With knowledge of the true data generating process at hand, we measure the sacrifice ratio as follows

$$SR = \frac{1}{\bar{\pi}^H - \bar{\pi}^L} \sum_{t=T^a}^N x_t, \quad (19)$$

where the denominator $\bar{\pi}^H - \bar{\pi}^L$ corresponds to the change in the inflation target as opposed the change in a statistical measure of trend inflation. The output gap is the model measure, x_t , which is the deviation of output from its efficient level as opposed to a deviation from a statistical measure of trend. T^a is the time the policy is announced, and N corresponds to the end of the disinflation episode. The end of the disinflation episode in a stochastic simulation is not as clear cut as in the deterministic case. In non-a stochastic simulation, a disinflation episode is defined as completed when the output gap returns to zero. In a stochastic simulation, though, this definition is undesirable because shocks may close the output gap before inflation reaches its new target. Therefore, we define the end of disinflation episode based on the path of inflation. We take the first quarter in which inflation is within two percentage points of the central bank's new inflation target ($|\pi^L - \pi_t| < 2\%$) to be the end of the disinflation episode, which is consistent with the two percent threshold of [Ball \(1994\)](#).²² This assumption yields a distribution of the sacrifice ratio under rational expectations that is centered on the sacrifice ratio implied by the non-stochastic simulation, where the end of the disinflation is determined by the output gap returning to zero.

5.1 Conditional Distributions of the Sacrifice Ratio

We now illustrate how credibility, anticipation, the degree of gradualism and the size of the disinflation shape the distribution of the sacrifice ratio. For these exercises, we use Monte Carlo simulations of disinflations at the US parameter values given in [Table 3](#)

²²Our results are robust to small changes in this cutoff. Shrinking the band increases cost estimates and expanding the band decreases cost estimates.

with the initial inflation target set to 12% and the gain parameter, γ , to 0.02.²³

Figure 3 shows the distribution of sacrifice ratio under full credibility ($\lambda = 1$), imperfect credibility ($\lambda = 0.5$), and no credibility ($\lambda = 0$) for an unanticipated cold turkey disinflation of 10%. Consistent with the findings in the existing literature, the mean sacrifice ratio increases on average with lower credibility. However, the effect on the mean sacrifice ratio does not fully reflect the effect of credibility on the distribution. The primary effect of low credibility is to increase variance and the right tail of the distribution which is generally reflected in increased skewness. Although high credibility significantly lowers the chance of costly disinflations, low credibility does not rule them out. All values of λ are potentially consistent with costless disinflations.

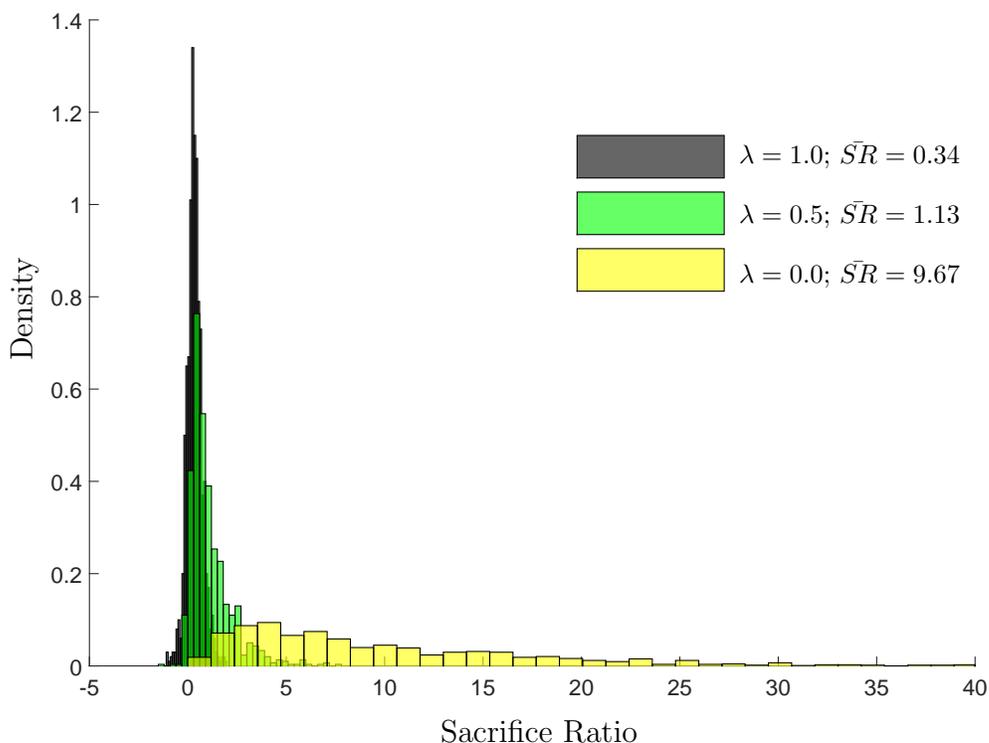
The fact that costless disinflations are consistent with low credibility is an insight that is obtained from studying the distribution. The literature has shown that the conditional mean changes with credibility but has abstracted from the impact on higher order moments. This impact that credibility has on the variance and right tail of the distribution applies to all disinflation policies to varying degrees.

A policy strategy to reduce the sacrifice ratio is to pre-announce the disinflation. Recall that in the simple model of Section 4, under perfect foresight, this policy creates a boom in output, a negative sacrifice ratio. Figure 4 compares the distribution of the sacrifice ratio for an anticipated disinflation with an unanticipated disinflation when the central bank lacks full credibility ($\lambda = 0.5$) for the same policy studied in Figure 3. In the anticipated case, the change in the inflation target is announced two quarters in advance. Consistent with the analytical results, when the disinflation is anticipated the distribution of the sacrifice ratio shifts to the left, which generally results in lower sacrifice ratios. But the impact on the mean is modest.

Table 4 gives estimates of the first four moments of the distribution to show how credibility interacts with pre-announcing the policy. As credibility increases, the mean

²³The gain is close to the value estimated by Milani (2007) on US data.

Figure 3: Credibility and the distribution of the sacrifice ratio



Notes: Distribution of the sacrifice ratio for a 10% disinflation under the US calibration. \bar{SR} is the mean sacrifice ratio.

sacrifice ratio falls and the distribution skews more to the left. A key takeaway is that pre-announcing a disinflation remains beneficial even when credibility is low. This is because the announcement triggers a partial fall in inflation expectations, which pushes inflation below the old inflation target, allowing the central bank to lower interest rates and support aggregate demand. The corresponding fall of inflation feeds back onto agents' belief through adaptive learning, generating faster convergence of inflation expectations towards the new inflation target and mitigating output costs.

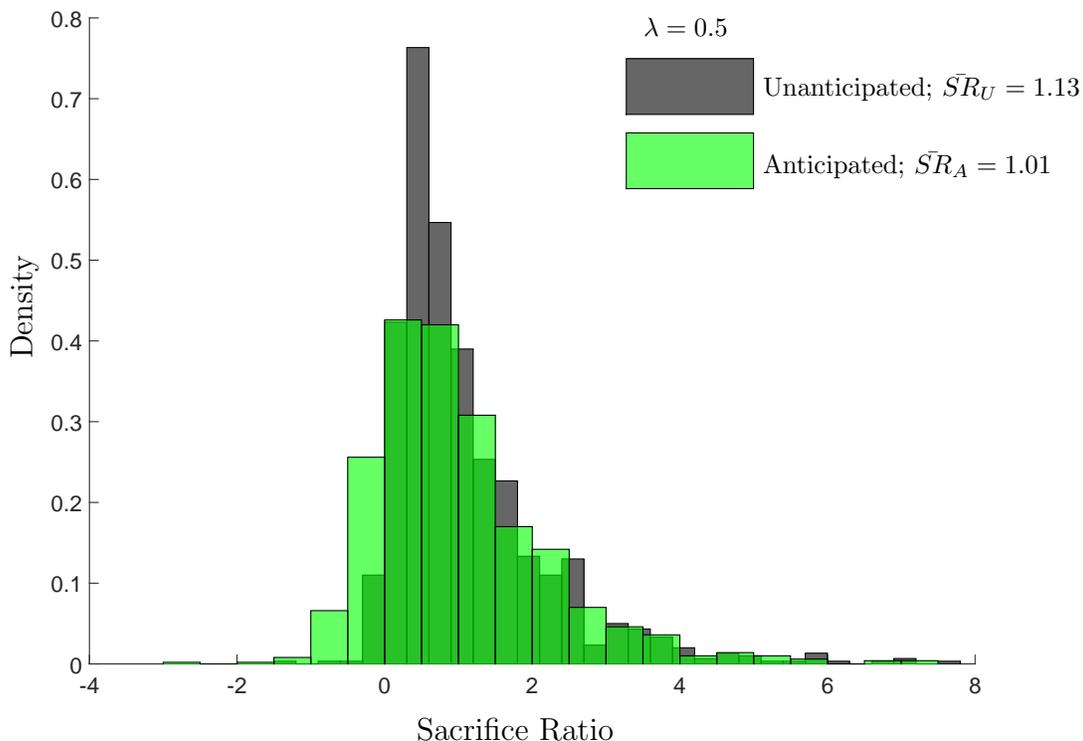
Another strategy proposed in the literature to lower the sacrifice ratio is gradualism. Figure 5 compares the distribution of the sacrifice ratio for a gradual disinflation to a cold turkey disinflations. We set $\lambda = 0.5$. Given the time-varying autoregressive specification for the short-run inflation target, $\bar{\pi}_t = (1 - \rho_\pi^*)\bar{\pi} + \rho_\pi^*\bar{\pi}_{t-1}$, the central bank's choice of

Table 4: Policy comparisons

| | $\lambda = 1/3$ | $\lambda = 2/3$ | $\lambda = 1$ |
|---------------|-----------------|-----------------|---------------|
| Unanticipated | | | |
| Mean: | 2.38 | 0.57 | 0.35 |
| St. Dev: | 2.04 | 0.62 | 0.39 |
| Skewness: | 1.93 | 3.10 | 0.18 |
| Kurtosis: | 8.47 | 19.77 | 4.55 |
| Anticipated | | | |
| Mean: | 2.33 | 0.42 | -0.09 |
| St. Dev: | 2.09 | 0.88 | 0.37 |
| Skewness: | 1.76 | 1.69 | -0.51 |
| Kurtosis: | 7.90 | 9.28 | 9.21 |
| Gradual | | | |
| Mean: | 1.16 | 0.21 | -0.14 |
| St. Dev: | 1.35 | 0.97 | 0.80 |
| Skewness: | 0.82 | 0.01 | -1.47 |
| Kurtosis: | 4.61 | 6.09 | 7.80 |

Notes: This table compares the moments of the predicted distribution of the sacrifice ratio for a 10% disinflation using the US calibration of the model for different types of policies and different levels of credibility.

Figure 4: Anticipated vs. unanticipated disinflations

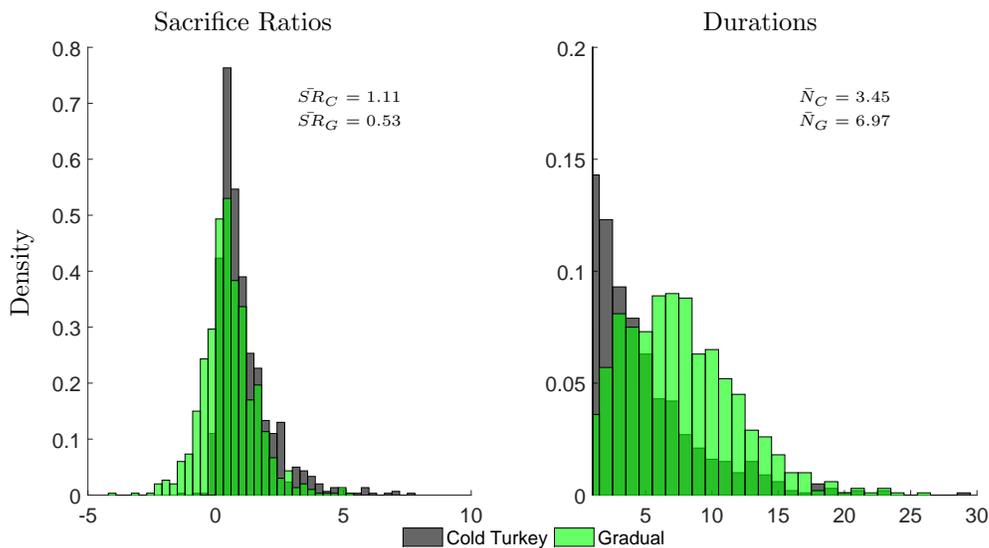


Notes: Distribution of the sacrifice ratio for a 10% disinflation under the US calibration. The anticipated disinflation is announced two quarter prior to implementation. $\bar{S}R_U$ is the mean sacrifice ratio of unanticipated disinflations and $\bar{S}R_A$ is the mean sacrifice ratio of anticipated disinflations.

ρ_π^* governs the degree of gradualism. The gradual disinflation we consider sets $\rho_\pi^* = 0.8$, which implies the short-run inflation target transitions from 12% to 2% in just over 8 quarters. A gradual disinflation is inherently less costly because the central bank responds less aggressively. This is because the short-run inflation target is closer to actual inflation when the policy is announced. The trade-off, though, is that gradual disinflations last longer.

Table 4 shows how this policy interacts with credibility. Once again, increases in credibility lower the mean sacrifice ratio and decrease right skewness. The effectiveness of gradual policies suggests a relationship between the cost of the disinflation and the size of the change in the inflation target. Figure 6 compares the distribution of the sacrifice

Figure 5: Gradual versus cold turkey disinflations



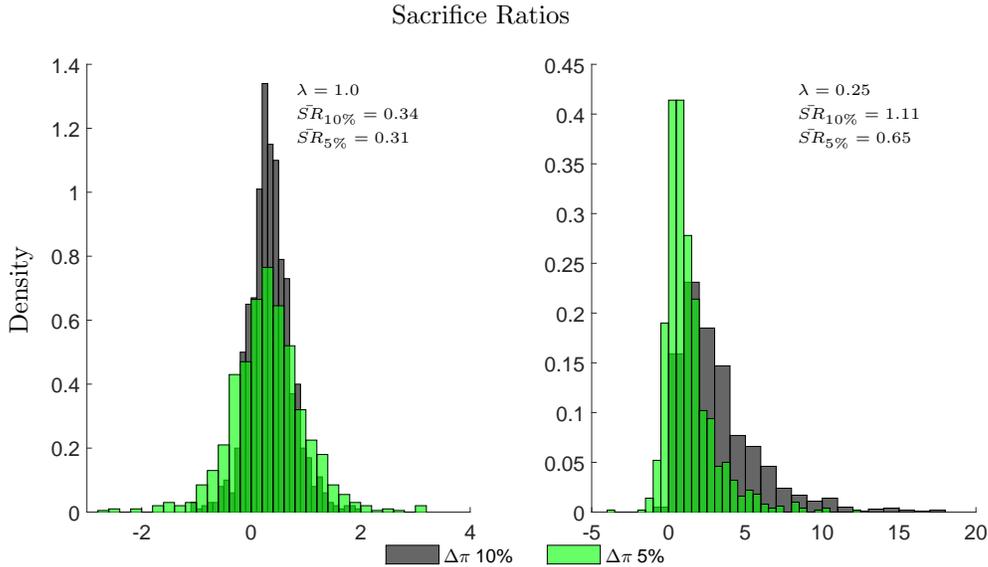
Notes: Distribution of the sacrifice ratio and the distribution of the duration of the disinflation episodes for a 10% disinflation under the US calibration for a gradual ($\rho_\pi^* = 0.8$) and a cold turkey disinflation. $\bar{S}R_C$ is the mean sacrifice ratio of cold turkey disinflations and $\bar{S}R_G$ is the mean sacrifice ratio of gradual disinflations. \bar{N}_C is the mean duration in quarters of cold turkey disinflations and \bar{N}_G is the mean duration in quarters of gradual disinflations.

ratio for two different size disinflations under full credibility and imperfect credibility ($\lambda = 0.25$).

With full credibility, the size of the disinflation has a modest impact on the distribution of the sacrifice ratio. With imperfect credibility, the larger the disinflation is, the more costly it is on average and the more right skewed the distribution becomes. Recalling the simple model of Section 4 again, the output cost of a disinflation is proportional to the change in the inflation target scaled by credibility. In this case, large changes in the inflation target do not move inflation expectations much when the policy is announced. Therefore, lower inflation will be implemented through actual realizations of lower inflation. The actual realizations of low inflation can happen by luck or by an aggressive response of the central bank. The interplay of luck and the response of the central bank generate the long right tail of the distribution.

Overall, the design of a disinflation policy has small effects on the mean of the sacrifice ratio. For all levels of credibility, the main benefit of pre-announcing the disinflation

Figure 6: Size of the disinflation



Notes: Distribution of the sacrifice ratio for a 10% and 5% disinflation under the US calibration. $\bar{S}R_{10\%}$ is the mean sacrifice ratio of 10% disinflations and $\bar{S}R_{5\%}$ is the mean sacrifice ratio of 5% disinflations.

or gradually reaching the new inflation target lies in reducing the right tail of the distribution.

5.2 The sacrifice ratio of opportunistic disinflations

The distribution highlights that shocks are important determinants of the realized cost of a disinflation. Our analysis up to this point, however, assumes that the disinflation is carried out regardless of the state of the economy. If inflation expectations depend on the recent history of the data, as in our model, then a central bank may consider implementing a disinflation following a period of falling inflation. In fact, [Bomfim and Rudebusch \(2000\)](#) note that the Federal Reserve discussed opportunistic disinflation strategies in FOMC meetings in the late 1980's and the approach is consistent with the behavior of many central banks who took advantage of the low inflation rates of the 1990's to implement inflation targeting.²⁴

²⁴See [Rasche and Williams \(2007\)](#) for empirical examples of these types of policy. See [Aksoy et al. \(2006\)](#) for a formulation of an opportunistic Taylor-type rule in an estimated sticky price model.

Table 5: The sacrifice ratio and luck

| | US Calibration: Small Shocks | | | Argentinean Calibration: Large Shocks | | |
|-----------------|------------------------------|---------------------------|----------------------|---------------------------------------|---------------------------|----------------------|
| | Inflation Below Target | Inflation Above Target | Full Distribution | Inflation Below Target | Inflation Above Target | Full Distribution |
| $\lambda = 1/3$ | | | | | | |
| Mean | 0.92 | 2.59 | 2.35 | 0.10 | 7.54 | 3.59 |
| St. Deviation | 0.97 | 2.07 | 2.02 | 5.04 | 9.16 | 7.05 |
| Skewness | 1.21 | 1.61 | 1.67 | 0.51 | 2.85 | 2.62 |
| Kurtosis | 8.09 | 5.83 | 6.84 | 13.59 | 11.8 | 12.34 |
| N | 1,200 | 1,211 | 5,000 | 1,123 | 1,110 | 5,000 |
| $\lambda = 2/3$ | | | | | | |
| Mean | 0.23 | 0.61 | 0.54 | -1.30 | 4.24 | 1.59 |
| St. Deviation | 0.36 | 0.62 | 0.58 | 5.64 | 5.96 | 5.35 |
| Skewness | 1.35 | 2.54 | 2.68 | -0.72 | 3.40 | 0.89 |
| Kurtosis | 18.47 | 10.5 | 15.62 | 8.37 | 17.97 | 12.06 |
| N | 1,194 | 1,189 | 5,000 | 1,120 | 1,097 | 5,000 |
| $\lambda = 1$ | | | | | | |
| Mean | 0.12 | 0.37 | 0.34 | -2.54 | 3.79 | 0.73 |
| St. Deviation | 0.53 | 0.38 | 0.40 | 7.31 | 5.94 | 6.77 |
| Skewness | 0.26 | 0.33 | -0.05 | -0.30 | 0.79 | -0.83 |
| Kurtosis | 6.39 | 7.01 | 6.65 | 6.97 | 13.76 | 9.4 |
| N | 1,178 | 1,173 | 5,000 | 1,108 | 1,094 | 5,000 |

Notes: The mean, standard deviation, skewness, and kurtosis refers to the distribution of the sacrifice ratio for disinflation policies implemented after periods for which average inflation is below or above target. We define average inflation as being 'below or above target' if the deviation of $1/4 \sum_{t=T^*-4}^{T^*-1} \pi_t$ is more than 1% with small shocks and more than 5% with the large shock.

To investigate the quantitative gains to an opportunistic disinflation, we simulate 5,000 disinflation episodes and compute sacrifice ratios for cases where inflation is either above or below target on average over the four quarters prior to the policy change.

Table 5 reports the mean, standard deviation, and skewness of the simulated distribution for disinflations of 10% and 30% for the US and Argentinean calibrations, respectively. Column 1 reports the results for average inflation 1% below target, column 2 reports the results for average inflation 1% above target, and column 3 reports the results from all 5,000 simulated disinflations. Columns 4, 5, and 6 report results with the Argentinean calibration. Because shocks are larger in this case, we consider 5% below or above the prevailing target as the cutoff.

Implementing a disinflation following a period when trend inflation is below the current inflation target lowers the sacrifice ratio at all levels of credibility. There are, how-

Table 6: Determinants of the sacrifice ratio

| VARIABLES | Model-implied measure | | | | Ball measure | | | |
|--|-----------------------|-------------------|-------------------|-------------------|--------------------|--------------------|--------------------|--------------------|
| | SR | SR | SR | SR | SR | SR | SR | SR |
| | $\lambda = 1$ | $\lambda = 1/2$ | $\lambda = 0$ | Mixed | $\lambda = 1$ | $\lambda = 1/2$ | $\lambda = 0$ | Mixed |
| Change Trend Inflation ($\Delta\pi$) | 0.11*** (0.02) | 0.05** (0.02) | 1.19*** (0.10) | 0.51*** (0.04) | -0.48*** (0.04) | -0.57*** (0.06) | -0.57*** (0.10) | -0.48*** (0.04) |
| Peak Trend Inflation | 0.13*** (0.03) | 0.09*** (0.03) | 0.87*** (0.17) | 0.61*** (0.05) | -0.07* (0.04) | -0.10* (0.04) | 0.04 (0.06) | -0.04 (0.03) |
| Duration | 0.07*** (0.02) | 0.20*** (0.01) | 0.23*** (0.02) | 0.33*** (0.01) | 0.37*** (0.02) | 0.38*** (0.02) | 0.37*** (0.03) | 0.34*** (0.02) |
| N | 334 | 333 | 333 | 1,000 | 982 | 1,012 | 1,016 | 3,010 |
| R-squared | 0.07 | 0.57 | 0.69 | 0.68 | 0.22 | 0.20 | 0.14 | 0.15 |

Notes: Regressions on simulated data. Estimated standard errors are reported in parentheses. *** significant at 1%; ** significant at 5%; * significant at 10%.

ever, more significant gains to an opportunistic policy with low credibility. In particular, an opportunistic disinflation unambiguously lowers the variance and skewness of the distribution of the sacrifice ratio, reducing the chance of a very costly outcomes. The reductions in the mean sacrifice ratio are large enough that opportunism often trumps credibility. For example, in the Argentinean calibration with $\lambda = 1/3$, the mean sacrifice ratio of an opportunistic disinflation (0.10) is smaller than the mean sacrifice ratio obtained for the whole distribution with perfect credibility (0.73). These results provide a strong rationale for the approach taken in practice by many central banks of implementing inflation targeting after a period of below trend inflation.²⁵ Therefore, a policymaker operating in an environment of low credibility should take advantage of periods of lower inflation that occur by chance to implement lower inflation targets.

5.3 Sacrifice ratio regressions on simulated data

Next, we use the quantitative model to reexamine the regression results found in empirical studies of the sacrifice ratio. We simulate in total 1,000 unanticipated cold-turkey disinflations using the US parameterization of the model: one third for $\lambda = 1$, one third

²⁵The comparison above is done at the same value of the gain parameter for both calibrations. In reality, it is reasonable to expect a higher gain in countries like Argentina, which are exposed to more frequent regime changes. Because a higher gain implies that the most recent observations carry more weight, a higher gain makes the opportunistic approach to disinflation even more desirable.

for $\lambda = 1/2$, and one third for $\lambda = 0$. We illustrate the issues by focusing on the coefficient on $\Delta\pi$. We therefore randomize over the size of the disinflation, $\Delta\pi$, while holding all other characteristics of the disinflation constant.²⁶

We calculate two measures of the sacrifice ratio: the model-implied sacrifice ratio, as given by Equation (19), which relies on the intended change of the inflation target and the model output gap, and the empirical measure of the sacrifice ratio, given by Equation (1), which relies on the statistical filters described in Section 2 for computing the change in trend inflation and the output gap. We then run the exact same regressions as in Table 1 on both measures of the sacrifice ratio.

Table 6 shows regression results for three cases and for the case where the 1000 disinflations are pooled together. The regressions for the model-implied measure of the sacrifice ratio are on the left and those for the empirical measure are on the right.

The regression estimates for the empirical measure of the sacrifice ratio correspond well to those computed using actual data. There is a negative correlation between the change in inflation and the peak of trend inflation with the sacrifice ratio, and a positive correlation with the duration as in the cross country data. The point estimates are close to those in Table 1, despite the fact that the actual data come from 42 different countries. The R-squared of these regressions are relatively low as is the case in the data as well.

The regression results using the empirical measure of the sacrifice ratio, however, stand in contrast to the model implied sacrifice ratio results. The coefficient estimates for the three determinants of the sacrifice ratio are positive and the R-squared are higher especially when disinflations under low credibility are included in the sample. The change in sign and magnitudes between the two cases may seem surprising. For example, a central bank with little credibility deciding on the size of a disinflation would believe that a larger disinflation is less costly. With the model, however, we know this is not the case.

²⁶The size of the disinflation ranges from 2% to 22% and is drawn from a Chi-squared distribution with five degrees of freedom to approximate the changes in the data. The mean change is 7.1%.

There are three reasons why the statistical filters used in empirical studies distort the coefficient estimates. One reason relates to the sample sizes in the two cases. The sample size is different depending on which measure of the sacrifice ratio is used. Under the model-implied measure, there are exactly 1,000 disinflations, while using Ball's measure we find 3,000 disinflations. This discrepancy arises because Ball's measure defines any fall in trend inflation of 2% or more to be a disinflation episode. As such, the measure picks up the intended disinflations as well as unintended ones. In some cases, Ball's measure may even miss intended disinflation for which adverse shocks prevent inflation from falling enough.

The second reason is that Ball's measure of the output gap assumes that output is at trend at the start of the disinflation and again four quarters after the end of the disinflation. This assumption underestimates the cost of disinflation precisely because the output gap is required to close by construction at the end of the disinflation. In our model, when credibility is low, negative output gaps can often persist for some time once inflation is close to the new target. The third reason is that the statistical measure of trend inflation defines any trough as the end of a disinflation. These troughs are often due to shocks and may be far above the new inflation target. This means some disinflation episodes are measured to be smaller than the central bank actually intended. These filters therefore introduce a negative correlation between the size of the disinflation and the sacrifice ratio, which is not predicted by the structural model.

The model implies that when the central bank is credible, the sacrifice ratio should be close to zero regardless of the change in the inflation target. In the simple model the output gap at the time of the disinflation is given by, $x_{T^*} = (1 - \lambda) \frac{(\pi^L - \pi^H)(\alpha - 1)}{(1 + \alpha\psi)}$, which shows that the closer λ is to unity, the less explanatory power the change in inflation, $(\pi^L - \pi^H)$, would have on the sacrifice ratio. Although this expression does not hold exactly in the larger model because of endogenous persistence, it is still the case that there is little variation in the sacrifice ratio as a function of the size of the disinflation,

$\Delta\pi$, for high values of λ . In other words, the mean of the sacrifice ratio conditional on the size of the disinflation can be expected to be the same for all size disinflations when credibility is high. In contrast, when λ is closer to zero, the change in the inflation target can be expected to have more explanatory power.

Consistent with the analytical expressions, when $\lambda = 1$, the coefficient on $\Delta\pi$ is close to zero (0.11) and the R-squared is low (0.07). When $\lambda = 0$, the coefficient on $\Delta\pi$ (1.19) and the R-squared (0.69) increase. But when the data is pooled, the coefficient estimates become weighted averages of the three cases which suggest a fully credible policymaker will overestimate the cost of disinflation, while the less credible one will underestimate it.

We find that the standard regressions may be misleading for policymakers contemplating the costs of disinflation. The model suggests that the statistical filters used to measure disinflations and the output gap introduce noise in the data and bias coefficient estimates. Importantly, whether or not one sees a strong relationship between the sacrifice ratio and observed characteristics of a disinflation depends nonlinearly on the credibility of the central bank.

6 Conclusion

We propose a framework to model imperfectly credible announcements and apply it to study different types of disinflation policies. We study not just the mean of the sacrifice ratio, which has been the focus of the literature, but the distribution of the sacrifice ratio. A lower degree of credibility increases the mean sacrifice ratio of a given disinflation. But credibility has an impact that goes well beyond the mean, affecting higher order moments of the distribution.

Our framework allows us to quantify the gains to an opportunistic approach to disinflation. We find these to be significant providing a rationale for the approach taken in

practice by many central banks. We then use the model to sharpen the interpretation of reduced-form regressions of the sacrifice ratio. The coefficients in these regressions are likely to be misleading for policymakers calibrating a disinflation. The coefficients that one obtains pooling all data are weighted averages of the coefficients of various conditional distributions and as such may not reflect the costs faced by a given policymaker. More importantly, perhaps, is that the statistical filters to compute sacrifice ratios in these empirical studies give rise to biases in the estimates.

Disinflations raise the more general question of what are the implications of permanent changes in the rate of trend inflation. This is important because some economists, motivated by the zero lower bound, have called for raising inflation targets. Critics of this proposal have emphasized that the output cost of a possible reversal in the future may be high, in line with our results.²⁷ Our analysis of disinflations would also apply to increases of the inflation target but only to the extent that the zero lower bound is not a binding constraint. Studying changes in trend inflation in the context of imperfectly anchored expectations, but accounting for the zero lower bound, is an exciting project that we leave for future research.

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²⁷See [Blanchard et al. \(2010\)](#) and the open letter signed by a number of prominent economists in November 2010 asking Federal Chairman Ben Bernanke to reconsider the use of quantitative easing (<http://blogs.wsj.com/economics/2010/11/15/open-letter-to-ben-bernanke/>).

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Appendix A

Data and estimation: We do two estimations of the model. One with data from Argentina and one with the data from the United States. The model is estimated on output per capita growth, nominal money growth and inflation for Argentina for the period 2003q3 to 2014q1 using Bayesian methods. In preliminary attempts to estimate the model we used a 3-month nominal interest rate. The estimation routine in this case, however, consistently led to implausibly parameter values. The issue is that the unconditional mean of the interest rate, inflation and output and money growth do not satisfy the steady state relations. The model implies that the steady state level of the nominal interest rate satisfies $r = z\pi/\beta$. But in our sample, the mean of the nominal interest rate is 0.0198 and the sample means for inflation and output growth require $\beta > 1$. Because the mean growth rate of money growth, 0.0541, is closer to the sum of mean inflation, 0.0361, and mean output growth, 0.0137, the estimation on those three observables seem more reliable.

In the case of inflation, we use the official series up until 2007q4 and then use the series implied by Billion Price Project. We choose the post-default period as it entails a period of relatively little structural change. During this period inflation accelerated, but this can be captured with our structure as persistent changes in π_t^* . The results of the estimation are shown in the left panel of Table 3. For the numerical analysis, we set parameter values at the mode.

The US estimation uses PCE inflation, real GDP growth, and the effective federal funds rates as observables from 1984Q1 through 2008Q1. Data is stopped at 2008Q1 to avoid the Financial Crisis and issues with incorporating the zero lower bound of nominal interest rates. Estimate results and priors are given in the right panel of Table 3. For the numerical analysis, we set parameter values at the mode.

Table A1: Disinflation episodes and sacrifice ratios

| Country | Date of DE | Peak Trend Inflation | Δ Inflation | Duration (Q) | Sacrifice Ratio | Country | Date of DE | Peak Trend Inflation | Δ Inflation | Duration (Q) | Sacrifice Ratio |
|-----------------|-----------------|----------------------|--------------------|--------------|-----------------|------------|-----------------|----------------------|--------------------|--------------|-----------------|
| Argentina* | 1962q4 : 1964q2 | 75.0 | 14.3 | 6 | 1.8 | Denmark | 1967q2 : 1969q4 | 8.9 | 4.1 | 10 | -0.2 |
| Argentina* | 1965q4 : 1969q2 | 87.8 | 63.6 | 14 | 0.9 | Denmark | 1974q3 : 1976q4 | 12.9 | 3.8 | 9 | 3.7 |
| Argentina* | 1972q2 : 1973q4 | 151.2 | 42.2 | 6 | 0.1 | Denmark | 1980q4 : 1987q1 | 12.0 | 8.2 | 25 | 0.1 |
| Argentina* | 1976q2 : 1977q4 | 543.4 | 216.4 | 6 | 0.0 | Denmark | 1988q4 : 1994q1 | 4.6 | 3.0 | 21 | 7.4 |
| Argentina* | 1984q3 : 1986q4 | 820.1 | 581.5 | 9 | 0.1 | Estonia | 2001q2 : 2003q4 | 4.9 | 2.9 | 10 | 0.2 |
| Argentina* | 1989q2 : 1997q1 | 1987.2 | 1986.5 | 31 | 0.1 | Estonia | 2007q4 : 2010q2 | 8.0 | 6.1 | 10 | 9.0 |
| Argentina | 1997q3 : 2000q1 | 3.1 | 7.0 | 10 | -4.2 | Finland | 1963q4 : 1966q3 | 7.5 | 3.1 | 11 | -4.4 |
| Argentina | 2003q1 : 2004q1 | 54.2 | 38.7 | 4 | -0.1 | Finland | 1967q4 : 1970q2 | 7.0 | 4.3 | 10 | 4.6 |
| Argentina | 2006q1 : 2008q4 | 31.3 | 16.4 | 11 | -3.1 | Finland | 1975q1 : 1979q2 | 17.3 | 9.5 | 17 | 6.2 |
| Argentina | 2011q1 : 2012q2 | 19.2 | 2.2 | 5 | -5.2 | Finland | 1981q2 : 1987q1 | 11.4 | 7.9 | 23 | -0.1 |
| Australia | 1975q1 : 1979q1 | 15.1 | 6.4 | 16 | -0.8 | Finland | 1989q4 : 1996q3 | 6.3 | 5.7 | 27 | 35.8 |
| Australia | 1982q3 : 1985q1 | 10.8 | 5.1 | 10 | 6.8 | Finland | 2001q1 : 2005q1 | 2.7 | 2.3 | 16 | 7.2 |
| Australia | 1989q2 : 1993q2 | 7.6 | 6.2 | 16 | 2.5 | Finland | 2007q4 : 2010q2 | 3.2 | 2.3 | 10 | 8.5 |
| Australia | 1995q3 : 1998q3 | 3.6 | 3.0 | 12 | 0.5 | France | 1963q1 : 1966q4 | 5.1 | 2.5 | 15 | -3.4 |
| Austria | 1974q4 : 1979q1 | 8.8 | 5.1 | 17 | 5.3 | France | 1975q1 : 1978q1 | 12.3 | 2.9 | 12 | 2.6 |
| Austria | 1981q2 : 1987q3 | 6.6 | 5.0 | 25 | -0.7 | France | 1981q2 : 1987q3 | 13.5 | 10.8 | 25 | 2.3 |
| Austria | 1993q1 : 1998q4 | 3.7 | 3.0 | 23 | 6.2 | Germany | 1973q4 : 1978q3 | 6.9 | 3.8 | 19 | 9.9 |
| Belgium | 1975q2 : 1979q2 | 12.6 | 7.9 | 16 | -0.1 | Germany | 1981q1 : 1987q2 | 5.9 | 5.7 | 25 | 6.4 |
| Belgium | 1982q3 : 1987q4 | 8.3 | 7.1 | 21 | 5.9 | Germany | 1992q3 : 1996q3 | 4.9 | 3.3 | 16 | 3.6 |
| Belgium | 1990q3 : 1999q1 | 3.4 | 2.4 | 34 | 29.0 | Greece | 1966q2 : 1968q4 | 4.2 | 3.3 | 10 | 4.9 |
| Brazil* | 1985q2 : 1986q3 | 214.2 | 35.6 | 5 | -0.1 | Greece | 1974q4 : 1978q1 | 21.2 | 8.9 | 13 | -8.1 |
| Brazil* | 1989q4 : 1992q2 | 2453.8 | 1735.3 | 10 | 0.0 | Greece | 1980q4 : 1984q4 | 24.4 | 5.8 | 16 | -2.5 |
| Brazil* | 1993q4 : 1998q4 | 2160.7 | 2156.8 | 20 | 0.0 | Greece | 1985q4 : 1989q1 | 20.8 | 7.0 | 13 | 3.3 |
| Brazil | 2002q4 : 2007q2 | 11.1 | 7.1 | 18 | 5.1 | Greece | 1991q1 : 2000q2 | 19.7 | 16.8 | 37 | 11.2 |
| Canada | 1975q1 : 1977q3 | 10.7 | 2.8 | 10 | 1.2 | Hungary* | 1991q1 : 1994q2 | 31.0 | 9.9 | 13 | 0.5 |
| Canada | 1981q3 : 1985q4 | 11.7 | 7.7 | 17 | 8.7 | Hungary | 1995q4 : 2003q2 | 25.3 | 20.2 | 30 | 3.1 |
| Canada | 1990q3 : 1994q2 | 5.3 | 4.3 | 15 | 13.2 | Iceland | 1964q1 : 1968q4 | 15.4 | 13.5 | 19 | -10.7 |
| Chile* | 1974q4 : 1982q2 | 487.1 | 471.2 | 30 | 1.0 | Iceland | 1980q4 : 1981q4 | 55.6 | 4.5 | 4 | -0.2 |
| Chile* | 1984q3 : 1988q4 | 25.7 | 9.2 | 17 | 20.7 | Iceland | 1983q1 : 1987q3 | 65.9 | 45.6 | 18 | 1.2 |
| Chile | 1990q3 : 2004q4 | 23.7 | 21.9 | 57 | -24.6 | Iceland | 1988q4 : 1995q3 | 23.2 | 21.6 | 27 | 2.3 |
| Chile | 2008q1 : 2010q3 | 6.4 | 5.4 | 10 | 5.1 | Iceland | 2001q2 : 2003q4 | 6.2 | 3.6 | 10 | -0.3 |
| China* | 1988q3 : 1991q2 | 17.6 | 14.1 | 11 | -1.8 | Iceland | 2009q1 : 2012q1 | 11.8 | 7.4 | 12 | 9.7 |
| China* | 1994q3 : 1999q2 | 20.8 | 21.8 | 19 | -11.8 | India* | 1967q1 : 1969q3 | 11.6 | 10.3 | 10 | -0.3 |
| China* | 2007q4 : 2010q1 | 5.0 | 3.5 | 9 | -11.2 | India* | 1974q2 : 1976q4 | 22.6 | 23.8 | 10 | 0.0 |
| Colombia* | 1974q3 : 1975q4 | 24.7 | 3.1 | 5 | 0.3 | India* | 1981q1 : 1985q3 | 12.0 | 5.3 | 18 | -0.7 |
| Colombia* | 1977q1 : 1979q2 | 26.6 | 5.0 | 9 | -3.3 | India* | 1991q3 : 1994q1 | 12.9 | 4.6 | 10 | 2.0 |
| Colombia* | 1980q4 : 1984q2 | 27.0 | 8.7 | 14 | -10.0 | India | 1997q4 : 2000q4 | 10.1 | 6.8 | 12 | -2.0 |
| Colombia* | 1991q2 : 2006q2 | 29.5 | 24.8 | 60 | -50.3 | India | 2009q4 : 2012q1 | 11.3 | 2.2 | 9 | -8.5 |
| Colombia | 2008q1 : 2010q4 | 6.3 | 3.6 | 11 | 7.0 | Indonesia* | 1974q1 : 1978q2 | 34.3 | 24.8 | 17 | 0.6 |
| Czech Republic* | 1992q4 : 1996q4 | 16.0 | 7.6 | 16 | 0.5 | Indonesia* | 1980q2 : 1982q4 | 17.7 | 7.5 | 10 | -1.8 |
| Czech Republic | 1997q3 : 2000q2 | 9.7 | 6.5 | 11 | 4.2 | Indonesia* | 1983q4 : 1986q2 | 11.0 | 5.3 | 10 | -0.9 |
| Czech Republic | 2001q1 : 2003q3 | 4.3 | 3.3 | 10 | 2.6 | Indonesia | 1998q3 : 2000q4 | 37.9 | 31.6 | 9 | 0.6 |
| Czech Republic | 2008q1 : 2010q3 | 4.4 | 3.1 | 10 | 5.3 | | | | | | |

Notes: Sacrifice ratios calculated using the methodology proposed by Ball (1994). *Denotes that annual GDP growth was used to calculate output losses.

Table A2: Disinflation episodes and sacrifice ratios II

| Country | Date of DE | Peak Trend Inflation | Δ Inflation | Duration (Q) | Sacrifice Ratio | Country | Date of DE | Peak Trend Inflation | Δ Inflation | Duration (Q) | Sacrifice Ratio |
|---------------|-----------------|----------------------|--------------------|--------------|-----------------|-----------------|-----------------|----------------------|--------------------|--------------|-----------------|
| Indonesia | 2001q4 : 2004q3 | 11.4 | 4.8 | 11 | 0.5 | Portugal | 2007q1 : 2010q1 | 2.8 | 2.4 | 12 | -2.7 |
| Indonesia | 2005q3 : 2010q4 | 11.3 | 6.5 | 21 | 0.7 | Russia | 1999q3 : 2007q1 | 58.0 | 48.4 | 30 | 0.6 |
| Ireland | 1981q2 : 1988q1 | 19.8 | 17.1 | 27 | 3.0 | Slovak Republic | 1994q1 : 1997q2 | 17.6 | 11.5 | 13 | -0.7 |
| Ireland | 2001q2 : 2005q1 | 5.2 | 2.9 | 15 | 3.9 | Slovak Republic | 2000q3 : 2002q2 | 11.1 | 5.6 | 7 | 0.9 |
| Ireland | 2007q3 : 2010q2 | 4.7 | 6.8 | 11 | 1.1 | Slovak Republic | 2003q4 : 2006q4 | 7.5 | 4.0 | 12 | 4.8 |
| Israel* | 1974q4 : 1976q4 | 37.9 | 5.4 | 8 | -1.5 | Slovak Republic | 2007q3 : 2010q2 | 3.7 | 2.2 | 11 | -7.9 |
| Israel* | 1984q4 : 1988q3 | 325.8 | 307.9 | 15 | 0.0 | Slovenia | 2001q1 : 2006q2 | 8.6 | 6.1 | 21 | 7.7 |
| Israel* | 1990q1 : 1993q4 | 18.6 | 7.9 | 15 | -1.0 | Slovenia | 2007q4 : 2010q3 | 4.4 | 3.0 | 11 | 3.1 |
| Israel* | 1994q1 : 2001q1 | 11.8 | 10.6 | 28 | -15.5 | South Africa | 1981q4 : 1984q1 | 15.0 | 2.8 | 9 | 7.7 |
| Israel | 2002q2 : 2004q3 | 3.7 | 4.0 | 9 | 3.2 | South Africa | 1986q2 : 1989q2 | 17.6 | 3.7 | 12 | -0.6 |
| Italy | 1963q4 : 1969q1 | 6.6 | 4.4 | 21 | 14.4 | South Africa | 1991q3 : 1997q3 | 14.9 | 7.4 | 24 | 8.4 |
| Italy | 1975q4 : 1978q4 | 17.7 | 4.3 | 12 | -2.1 | South Africa | 1997q4 : 2000q4 | 7.9 | 2.9 | 12 | 6.0 |
| Italy | 1980q4 : 1988q1 | 19.3 | 14.5 | 29 | 5.6 | South Africa | 2002q2 : 2005q1 | 8.1 | 7.6 | 11 | 1.4 |
| Italy | 1990q2 : 1994q2 | 6.4 | 2.1 | 16 | 4.1 | South Africa | 2008q3 : 2011q1 | 8.5 | 3.9 | 10 | 5.4 |
| Italy | 1995q2 : 1998q4 | 4.7 | 2.9 | 14 | 0.0 | Spain | 1965q3 : 1969q3 | 10.3 | 6.9 | 16 | 0.9 |
| Japan | 1974q3 : 1979q1 | 17.8 | 13.6 | 18 | 2.0 | Spain | 1977q2 : 1988q2 | 22.1 | 16.9 | 44 | 12.3 |
| Japan | 1980q4 : 1987q4 | 6.2 | 6.0 | 28 | 5.4 | Spain | 1990q1 : 1998q3 | 6.7 | 4.8 | 34 | 12.8 |
| Japan | 1990q4 : 1995q4 | 3.1 | 3.1 | 20 | 4.9 | Spain | 2007q3 : 2010q1 | 3.5 | 2.5 | 10 | -3.9 |
| Korea | 1970q3 : 1973q1 | 15.0 | 7.1 | 10 | 2.6 | Sweden | 1966q3 : 1969q1 | 5.7 | 3.2 | 10 | 5.0 |
| Korea | 1975q1 : 1977q4 | 24.4 | 12.8 | 11 | 0.0 | Sweden | 1977q2 : 1979q1 | 11.1 | 2.1 | 7 | 2.3 |
| Korea | 1980q3 : 1984q4 | 24.7 | 22.3 | 17 | 1.2 | Sweden | 1980q4 : 1987q2 | 12.5 | 8.2 | 26 | 9.2 |
| Korea | 1991q1 : 1996q4 | 8.8 | 4.2 | 23 | -0.5 | Sweden | 1990q4 : 1998q1 | 9.5 | 9.4 | 29 | 13.2 |
| Korea | 1997q4 : 2000q2 | 5.9 | 4.1 | 10 | 14.7 | Sweden | 2007q4 : 2010q1 | 2.7 | 2.1 | 9 | 10.1 |
| Latvia | 2008q1 : 2010q4 | 12.4 | 11.3 | 11 | 6.2 | Switzerland | 1966q3 : 1969q3 | 4.4 | 2.1 | 12 | 17.7 |
| Luxembourg | 1975q3 : 1979q1 | 10.6 | 6.6 | 14 | 1.1 | Switzerland | 1974q2 : 1978q1 | 9.2 | 8.0 | 15 | 8.3 |
| Luxembourg | 1982q3 : 1987q3 | 9.0 | 9.0 | 20 | 3.9 | Switzerland | 1982q1 : 1987q2 | 5.9 | 4.8 | 21 | 9.4 |
| Luxembourg | 1992q3 : 1998q4 | 3.3 | 2.3 | 25 | 36.7 | Switzerland | 1991q1 : 1998q3 | 5.5 | 5.3 | 30 | 12.9 |
| Mexico | 1974q3 : 1975q4 | 19.5 | 3.5 | 5 | -0.5 | Turkey | 1962q4 : 1965q1 | 5.3 | 2.1 | 9 | -12.9 |
| Mexico | 1977q4 : 1979q2 | 23.5 | 5.1 | 6 | -0.1 | Turkey | 1967q1 : 1969q3 | 10.8 | 5.2 | 10 | 0.6 |
| Mexico | 1983q3 : 1985q2 | 85.9 | 23.5 | 7 | -0.3 | Turkey | 1974q4 : 1976q3 | 22.2 | 4.0 | 7 | 0.0 |
| Mexico | 1987q3 : 1994q1 | 127.0 | 118.1 | 26 | 0.2 | Turkey | 1979q4 : 1982q4 | 77.2 | 47.0 | 12 | 0.7 |
| Mexico | 1996q2 : 2007q1 | 34.6 | 30.9 | 43 | -3.2 | Turkey | 1985q1 : 1986q4 | 45.8 | 8.9 | 7 | 1.3 |
| Netherlands | 1981q2 : 1987q3 | 6.7 | 7.0 | 25 | 8.9 | Turkey | 1989q2 : 1990q2 | 65.4 | 3.6 | 4 | 0.9 |
| Netherlands | 2001q4 : 2006q1 | 3.6 | 2.2 | 17 | 17.2 | Turkey | 1995q2 : 1996q3 | 96.6 | 16.3 | 5 | -0.1 |
| New Zealand * | 1980q4 : 1984q2 | 16.3 | 8.8 | 14 | 0.3 | Turkey | 1997q3 : 2005q2 | 85.8 | 77.5 | 31 | 3.0 |
| New Zealand * | 1986q3 : 1993q1 | 15.4 | 14.2 | 26 | 4.8 | Turkey | 2007q4 : 2010q3 | 9.6 | 2.8 | 11 | 21.0 |
| New Zealand | 1995q4 : 1999q1 | 3.0 | 2.4 | 13 | 1.1 | United Kingdom | 1975q2 : 1978q4 | 20.7 | 9.7 | 14 | 3.4 |
| Norway | 1975q3 : 1979q2 | 10.6 | 4.0 | 15 | -2.1 | United Kingdom | 1980q3 : 1984q2 | 15.7 | 10.8 | 15 | 3.2 |
| Norway | 1981q3 : 1985q3 | 12.6 | 6.7 | 16 | 8.6 | United Kingdom | 1991q1 : 2001q1 | 7.2 | 6.2 | 40 | 15.1 |
| Norway | 1987q3 : 1994q1 | 8.1 | 6.2 | 26 | 15.5 | United States | 1974q4 : 1977q2 | 9.9 | 3.7 | 10 | 4.7 |
| Portugal | 1977q4 : 1980q4 | 26.5 | 8.8 | 12 | -0.4 | United States | 1980q3 : 1984q2 | 12.3 | 8.5 | 15 | 4.6 |
| Portugal | 1984q2 : 1988q1 | 26.2 | 16.1 | 15 | 2.6 | United States | 1990q2 : 1995q1 | 5.1 | 2.4 | 19 | 14.1 |
| Portugal | 1990q2 : 1999q2 | 13.2 | 10.8 | 36 | 7.3 | United States | 2007q3 : 2010q1 | 3.4 | 2.6 | 10 | 3.1 |

Notes: Sacrifice ratios calculated using the methodology proposed by Ball (1994). *Denotes that annual GDP growth was used to calculate output losses.