Disinflations in a Model of Imperfectly Anchored Expectations: Online Appendix

Christopher G. Gibbs* and Mariano Kulish†

August 12, 2017

*School of Economics, UNSW, christopher.gibbs@unsw.edu.au
†School of Economics, UNSW, m.kulish@unsw.edu.au

1 Imperfectly Anchored Expectations

Consider the class of linearized structural models of n equations of the form

$$y_t = \mathbf{\Gamma} + \mathbf{A} y_{t-1} + \mathbf{B} \hat{\mathbb{E}}_t y_{t+1} + \mathbf{D} \varepsilon_t, \tag{1}$$

where y_t is a $n \times 1$ vector of state and jump variables and ε_t is a $l \times 1$ vector of exogenous variables. Without loss of generality, we take the latter to be white noise.[1] We assume $\hat{\mathbb{E}}_t y_{t+1}$ is a linear combination of the form

$$\hat{\mathbb{E}}_t y_{t+1} = \lambda \mathbb{E}_t^S y_{t+1} + (1-\lambda) \mathbb{E}_t^R y_{t+1} \tag{2}$$

where $0 \leq \lambda \leq 1$ and $\mathbb{E}_t^S y_{t+1}$ is the structural forecasting function, which is equivalent to rational expectations when $\lambda = 1$, while $\mathbb{E}_t^R y_{t+1}$ is the forecasting function under adaptive learning.

Rational expectations

When $\hat{\mathbb{E}}_t y_{t+1} = \mathbb{E}_t^S y_{t+1}$, expectations are rational and the solution to Equation (1) is a VAR of the form

$$y_t = \mathbf{C} + \mathbf{Q} y_{t-1} + \mathbf{G} \varepsilon_t, \tag{3}$$

which is the McCallum (1983) minimum state variable (MSV) solution. We restrict attention to parameter values that yield uniquely bounded solutions as is typically done in the literature. Given that $\mathbb{E}_t \varepsilon_{t+1} = 0$, it follows from Equation (3) that the forecasting function for $\lambda = 1$ is

$$\mathbb{E}_t^S y_{t+1} = \mathbf{C} + \mathbf{Q} y_t. \tag{4}$$

Adaptive learning

To construct the forecasting function under adaptive learning, we follow Evans and Honkapohja (2001) and solve the model given by Equation (1) by assuming that agents understand the reduced form structure of the economy but do not know how it is parameterized. Agents are assumed to have a *perceived law of motion* (PLM) of the economy given by

$$y_t = \tilde{\mathbf{C}} + \tilde{\mathbf{Q}} y_{t-1} + \tilde{\mathbf{G}} \varepsilon_t, \tag{5}$$

consistent with the MSV solution. The form of the MSV may vary depending on information set assumptions and may exclude terms to prevent multicollinearity in real time learning.[2]

Agents parameterize the model by recursively estimating a VAR of the form of Equation (5) using observed past data. In particular, agents estimate the parameters using the recursive least squares algorithm with information up until time $t-1$. To coincide

[1] All matrices in Equation (1) conform to the specified dimensions. The method we develop here may be further generalized as in Binder and Pesaran (1995) to allow additional lags of y_t as well as expectations at different horizons and from earlier dates.

[2] See Evans and Honkapohja (2001) for a discussion of these issues.

with the timing of the rational expectations solution, however, we assume agents use information through time t in their forecasts.

Agents estimate the parameters of the PLM, $\xi_t' = \begin{pmatrix} \tilde{\mathbf{C}}_t, & \tilde{\mathbf{Q}}_t, & \tilde{\mathbf{G}}_t \end{pmatrix}$, recursively

$$\begin{aligned} \xi_t' &= \xi_{t-1}' + \gamma \mathbf{\Omega}_t^{-1} z_{t-1}(y_{t-1} - \xi_{t-1}' z_{t-1}) && (6) \\ \mathbf{\Omega}_t &= \mathbf{\Omega}_{t-1} + \gamma(z_{t-1} z_{t-1}' - \mathbf{\Omega}_{t-1}) && (7) \end{aligned}$$

where $z_t' = \begin{pmatrix} 1 & y_t' & \varepsilon_t' \end{pmatrix}$, $\mathbf{\Omega}_t$ is the variance covariance matrix of z_t, and γ is the gain parameter. As is common in literature, we restrict our attention to the case where γ is a constant $0 < \gamma < 1$. A constant gain is similar to assuming rolling window regressions, where more weight is placed on newer observations and older observations are down-weighted or dropped from the sample.

The forecasting functions under adaptive learning are time-varying and given by

$$\mathbb{E}_t^R y_{t+1} = \tilde{\mathbf{C}}_t + \tilde{\mathbf{Q}}_t y_t. \tag{8}$$

Substituting this expression into Equation (1) gives rise to the *actual law of motion* (ALM) for the economy[3]

$$y_t = (\mathbf{I} - \mathbf{B}\tilde{\mathbf{Q}}_t)^{-1}(\mathbf{\Gamma} + \mathbf{B}\tilde{\mathbf{C}}_t + \mathbf{A}y_{t-1} + \mathbf{D}\varepsilon_t). \tag{9}$$

which implies a T-mapping given by

$$\begin{aligned} \bar{\mathbf{C}}_t &= (\mathbf{I} - \mathbf{B}\tilde{\mathbf{Q}}_t)^{-1}(\mathbf{\Gamma} + \mathbf{B}\tilde{\mathbf{C}}_t) \\ \bar{\mathbf{Q}}_t &= (\mathbf{I} - \mathbf{B}\tilde{\mathbf{Q}}_t)^{-1}\mathbf{A} \\ \bar{\mathbf{G}}_t &= (\mathbf{I} - \mathbf{B}\tilde{\mathbf{Q}}_t)^{-1}\mathbf{D}, \end{aligned}$$

The fixed point of this mapping is the rational expectations equilibrium given by equation (3), that is $\bar{\xi} = \begin{pmatrix} \mathbf{C}, & \mathbf{Q}, & \mathbf{G} \end{pmatrix}$ which is the limit of the recursive learning algorithm under well-known regularity conditions.

Imperfectly anchored expectations

When $0 < \lambda < 1$, we assume that forecasts are a linear combination of equations (4) and (8), that is,

$$\begin{aligned} \hat{\mathbb{E}}_t y_{t+1} &= \lambda \mathbb{E}_t^S y_{t+1} + (1-\lambda)\mathbb{E}_t^R y_{t+1} \\ &= \lambda(\mathbf{C} + \mathbf{Q}y_t) + (1-\lambda)(\tilde{\mathbf{C}}_t + \tilde{\mathbf{Q}}_t y_t). \end{aligned}$$

The reduced form solution when expectations are given by Equation (2) is

$$y_t = \hat{\mathbf{C}}_t + \hat{\mathbf{Q}}_t y_{t-1} + \hat{\mathbf{G}}_t \varepsilon_t \tag{10}$$

[3]Assuming the relevant inverse matrices exist.

where

$$\begin{aligned}
\hat{\mathbf{C}}_t &= [\mathbf{I} - \mathbf{B}(\lambda\mathbf{Q} + (1-\lambda)\tilde{\mathbf{Q}}_t)]^{-1}(\mathbf{\Gamma} + \lambda\mathbf{BC} + (1-\lambda)\mathbf{B}\tilde{\mathbf{C}}_t) \\
\hat{\mathbf{Q}}_t &= [\mathbf{I} - \mathbf{B}(\lambda\mathbf{Q} + (1-\lambda)\tilde{\mathbf{Q}}_t)]^{-1}\mathbf{A} \\
\hat{\mathbf{G}}_t &= [\mathbf{I} - \mathbf{B}(\lambda\mathbf{Q} + (1-\lambda)\tilde{\mathbf{Q}}_t)]^{-1}\mathbf{D}.
\end{aligned}$$

The fixed point of the mapping is again $\bar{\xi} = \begin{pmatrix} \mathbf{C}, & \mathbf{Q}, & \mathbf{G} \end{pmatrix}$.

Note also that the reduced-form solution nests rational expectations and adaptive learning expectations as special cases. If all weight is placed on rational expectations or if all the weight is placed on adaptive learning, then the standard solutions are obtained.

Announcements and permanent changes in policy

To incorporate announcements regarding changes to policy rule parameters, we follow Cagliarini and Kulish (2013) and use the VAR representation solution of Kulish and Pagan (2017). Assume at time T^a policy makers announce a future change of the inflation target that will be implemented at time T^*. This corresponds to a future change in one of the structural parameters, so in general, agents will anticipate the structural equations to evolve as follows

$$\begin{aligned}
y_t &= \mathbf{\Gamma} + \mathbf{A}y_{t-1} + \mathbf{B}\mathbb{E}_t y_{t+1} + \mathbf{D}\varepsilon_t \quad \text{for} \quad t = T^a, T^a+1, \ldots, T^*-1 \\
y_t &= \mathbf{\Gamma}^* + \mathbf{A}^* y_{t-1} + \mathbf{B}^*\mathbb{E}_t y_{t+1} + \mathbf{D}^*\varepsilon_t \quad \text{for} \quad t = T^*, T^*+1, \ldots
\end{aligned}$$

Kulish and Pagan (2017) show that from the time of the announcement, T^a, until the time of implementation, T^*, the solution for y_t is a time-varying coefficients VAR of the form

$$y_t = \mathbf{C}_t + \mathbf{Q}_t y_{t-1} + \mathbf{G}_t \varepsilon_t. \tag{11}$$

Because the announcement is taken to be credible, it follows that $\mathbb{E}_t y_{t+1} = \mathbf{C}_{t+1} + \mathbf{Q}_{t+1} y_t$ which implies by undetermined coefficients the following equivalences

$$\mathbf{C}_t = (\mathbf{I} - \mathbf{B}\mathbf{Q}_{t+1})^{-1}(\mathbf{\Gamma} + \mathbf{B}\mathbf{C}_{t+1}) \tag{12}$$

$$\mathbf{Q}_t = (\mathbf{I} - \mathbf{B}\mathbf{Q}_{t+1})^{-1}\mathbf{A} \tag{13}$$

$$\mathbf{G}_t = (\mathbf{I} - \mathbf{B}\mathbf{Q}_{t+1})^{-1}\mathbf{D}. \tag{14}$$

Starting from the solution to the final regime, $\mathbf{Q}_{T^*} = \mathbf{Q}^*$, one chooses the sequence $\{\mathbf{Q}_t\}_{t=T^a}^{T^*-1}$ that satisfies Equation (13). With that sequence in-hand one can calculate the sequences implied by Equation (12) and Equation (14).

Expectations in the presence of an announcement when $0 < \lambda < 1$ are given by

$$\begin{aligned}
\hat{\mathbb{E}}_t y_{t+1} &= \lambda\mathbb{E}_t^S y_{t+1} + (1-\lambda)\mathbb{E}_t^R y_{t+1} \\
&= \lambda(\mathbf{C}_{t+1} + \mathbf{Q}_{t+1} y_t) + (1-\lambda)(\tilde{\mathbf{C}}_t + \tilde{\mathbf{Q}}_t y_t).
\end{aligned}$$

Substituting this expression into the Equation (1), the reduced form solution is

$$y_t = \hat{\mathbf{C}}_t + \hat{\mathbf{Q}}_t y_{t-1} + \hat{\mathbf{G}}_t \varepsilon_t \tag{15}$$

where

$$\begin{aligned}
\hat{\mathbf{C}}_t &= [\mathbf{I} - \mathbf{B}(\lambda\mathbf{Q}_{t+1} + (1-\lambda)\tilde{\mathbf{Q}}_t)]^{-1}(\mathbf{\Gamma} + \lambda\mathbf{B}\mathbf{C}_{t+1} + (1-\lambda)\mathbf{B}\tilde{\mathbf{C}}_t) \\
\hat{\mathbf{Q}}_t &= [\mathbf{I} - \mathbf{B}(\lambda\mathbf{Q}_{t+1} + (1-\lambda)\tilde{\mathbf{Q}}_t)]^{-1}\mathbf{A} \\
\hat{\mathbf{G}}_t &= [\mathbf{I} - \mathbf{B}(\lambda\mathbf{Q}_{t+1} + (1-\lambda)\tilde{\mathbf{Q}}_t)]^{-1}\mathbf{D}.
\end{aligned}$$

The reduced form now includes both backward ($\tilde{\mathbf{Q}}_t$) and forward-looking ($\mathbf{Q}_{t+1}$) information. The backward-looking information comes from the estimated coefficients from adaptive learning expectations and the forward-looking information comes from announcement of future changes. In this context, λ serves as a natural measure of credibility precisely because it governs the impact of an announcement. When $\lambda = 0$, an announcement would have no impact on the reduced-form and forecasting functions would respond only to past data.

1.1 E-stability with imperfectly anchored expectations

The expectations we propose weaken the E-stability conditions. Any rational expectations equilibrium that is E-stable is also stable under learning in our framework. In addition, any rational expectations equilibrium can be made stable under learning as long as expectations are sufficiently anchored, meaning λ is sufficiently large. The intuition is that as λ approaches one, the learning dynamics become less important for expectations. This result is summarized by the following proposition.

Proposition 1: *There exists $0 < \lambda < 1$ such that the MSV solution given by $\bar{\xi} = \begin{pmatrix} \mathbf{C}, & \mathbf{Q}, & \mathbf{G} \end{pmatrix}$ is E-stable.*

Proof: The proof follows Proposition 10.3 of EH. The T-map implied by combining expectations is

$$T\begin{pmatrix} \tilde{\mathbf{C}} \\ \tilde{\mathbf{Q}} \\ \tilde{\mathbf{G}} \end{pmatrix} = \begin{pmatrix} \chi^{-1}(\mathbf{\Gamma} + \mathbf{B}\lambda\mathbf{C} + \mathbf{B}(1-\lambda)\tilde{\mathbf{C}}) \\ \chi^{-1}\mathbf{A} \\ \chi^{-1}\mathbf{D} \end{pmatrix},$$

where $\chi = \mathbf{I} - \mathbf{B}(\lambda\mathbf{Q} + (1-\lambda)\tilde{\mathbf{Q}})$. Stability is determined by evaluating the eigenvalues of the mapping linearized around a fixed point of interest. The linearized mapping with respect to $\tilde{\mathbf{C}}$, $\tilde{\mathbf{Q}}$, and $\tilde{\mathbf{G}}$ and evaluated at the REE can be computed as

$$\begin{aligned}
\mathbf{DT_C}(\mathbf{C}, \mathbf{Q}) &= (\mathbf{I} - \mathbf{BQ})^{-1}\mathbf{B}(1-\lambda) \\
\mathbf{DT_Q}(\mathbf{Q}) &= [(\mathbf{I} - \mathbf{BQ})^{-1}\mathbf{A}(1-\lambda)]' \otimes [(\mathbf{I} - \mathbf{BQ})^{-1}\mathbf{B}].
\end{aligned}$$

The requirement for E-stability is that the eigenvalues of the matrices $\mathbf{DT_C}(\mathbf{C}, \mathbf{Q})$ and $\mathbf{DT_Q}(\mathbf{Q})$ have real part less than one. Consider

$$(1-\lambda)\mathbf{A}x = (1-\lambda)\xi x \equiv \hat{\xi}x,$$

where $\mathbf{A}$ is equal to either $(\mathbf{I}-\mathbf{BQ})^{-1}\mathbf{B}$ or $[(\mathbf{I}-\mathbf{BQ})^{-1}\mathbf{A}]' \otimes [(\mathbf{I}-\mathbf{BQ})^{-1}\mathbf{B}]$, $\hat{\xi}$ is a vector of eigenvalues associated with A, $\hat{\xi}_i$ is the real part of an eigenvalue element of $\hat{\xi}$, and x is the corresponding right eigenvector. The requirement for stability is $\hat{\xi}_i < 1$ for all i. Let ξ_i be the real part of the eigenvalue of the T-map in the case where $\lambda = 0$, which corresponds to the standard adaptive learning case. Assume $\xi_i > 1$, then it must be the case that $\xi_i \geq \hat{\xi}_i$ since $\hat{\xi}_i = (1-\lambda)\xi_i$. Pick λ such that $\lambda < 1 - \xi_i^{-1} < 1$. Now, since $\hat{\xi}_1 = (1-(1-\xi_1^{-1}))\xi_1 = 1$ and $1-\xi_i^{-1} < 1$ for all finite ξ_i, there always exists a λ such that $\hat{\xi}_i < 1$ for all i.

2 Analytical results on disinflations

This section illustrates our expectations framework in a New Keynesian model simple enough to allow for analytical results. We use this model to highlight the source of output gains and losses in response to a disinflation and to show how our measure of imperfect credibility changes outcomes relative to rational expectations. The model we consider is described by an IS-curve, a Phillips curve, and a Taylor-type monetary policy rule:

$$\begin{aligned} x_t &= \hat{\mathbb{E}}_t x_{t+1} - (r_t - \hat{\mathbb{E}}_t \pi_{t+1}) \\ \pi_t &= \beta \hat{\mathbb{E}}_t \pi_{t+1} + \psi x_t \\ r_t &= \bar{\pi} + \alpha (\pi_t - \bar{\pi}) . \end{aligned}$$

where x_t is the output gap, π_t is inflation, r_t is the nominal interest rate, and $\bar{\pi}$ is the inflation target. We set β to one so that in steady state $\pi_t = \bar{\pi}$, $r_t = \bar{\pi}$ and $x_t = 0$.

2.1 Origins of the sacrifice ratio

A disinflation policy is a permanent change in the inflation target, $\bar{\pi}$. The output cost depends on the path of inflation expectations. To see this, substitute the Phillips curve and monetary policy rule into the IS-curve and iterate forward:

$$x_t = (\alpha - 1)\sum_{j=1}^{\infty}(1+\alpha\psi)^{-j}(\bar{\pi} - \hat{\mathbb{E}}_t \pi_{t+j}), \tag{16}$$

where we have used the fact that in a stable equilibrium $\lim_{j\to\infty} \hat{\mathbb{E}}_t x_{t+j} = 0$. Equation (16) shows that the cost of disinflation depends on inflation expectations relative to the central bank's inflation target. To the extent that inflation expectations do not fall when the inflation target falls, that is $(\bar{\pi} - \hat{\mathbb{E}}_t \pi_{t+j}) < 0$, there is an output cost. The aggressiveness of monetary policy as captured by α and the degree of price stickiness as captured by ψ act only to scale the output cost. In particular, the more aggressive the central bank is in fighting deviations from its current inflation target, the larger the output cost.

The cost of a credible cold turkey disinflation under rational expectations is zero because $\mathbb{E}_t \pi_{t+j} = \bar{\pi}$ for all j. The only way to generate a non-zero sacrifice ratio under rational expectations is for the policy to be anticipated. As noted by Ball (1994), if the policy is anticipated, then expectations move before the change in the target. This causes either a boom or a slump in output depending on the aggressiveness of monetary policy. Output responds to an anticipated change in the target because the central bank temporarily defends its old target. The expectation of a decrease in the target causes expectations to fall, and hence actual inflation to fall relative to the old target, which causes monetary policy to reduce the nominal interest rate in response. The reduction leads to a fall in real interest rates if $\alpha > 1$, which stimulates aggregate demand and increases output.

2.2 Disinflations with imperfect credibility

With imperfect credibility, a disinflation program generates output losses because expectations do not immediately adjust to the new inflation target. The central bank, therefore, must contract demand to move inflation and inflation expectations to the desired level. An imperfectly credible disinflation, therefore, can be characterized as having two different phases: an initial impact which depends on the response of expectations to the announcement and a convergence towards the new target once the policy is implemented.

2.2.1 Initial impacts

Consider first an unanticipated cold turkey disinflation. The central bank lowers the inflation target from π^H to π^L. The inflation target is π^H until T^* when the central bank announces and implements π^L. Substituting the monetary policy rule into the output gap equation, the model can be reduced to the following two equations

$$\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = \frac{\alpha - 1}{1 + \alpha\psi} \begin{pmatrix} \bar{\pi} \\ \psi\bar{\pi} \end{pmatrix} + \frac{1}{1 + \alpha\psi} \begin{pmatrix} 1 & 1 - \alpha \\ \psi & 1 + \psi \end{pmatrix} \begin{pmatrix} \hat{\mathbb{E}}_t x_{t+1} \\ \hat{\mathbb{E}}_t \pi_{t+1} \end{pmatrix}$$

and written in the form of Equation (1) as

$$y_t = \mathbf{\Gamma} + \mathbf{B}\hat{\mathbb{E}}_t y_{t+1}, \tag{17}$$

where $\mathbf{A}$ and $\mathbf{D}$ are zero. In this case, the MSV solution, Equation (3), reduces to

$$y_t = \mathbf{C}$$

where $\mathbf{C} = (0, \bar{\pi})'$. Assuming the economy is at steady state prior to the disinflation, one can show that expectations at T^* are $\hat{\mathbb{E}}_t y_{t+1} = \lambda \mathbf{C}^L + (1-\lambda)\mathbf{C}^H$, where $\mathbf{C}^L = (0, \pi^L)'$ and $\mathbf{C}^H = (0, \pi^H)'$. The output gap and inflation in period T^* are given by the expressions below

$$\begin{aligned} x_{T^*} &= (1-\lambda)\frac{(\pi^L - \pi^H)(\alpha - 1)}{(1 + \alpha\psi)} \\ \pi_{T^*} &= (1-\lambda)\frac{(1+\psi)}{(1+\alpha\psi)}\pi^H + \frac{(\alpha-1)\psi + \lambda(1+\psi)}{(1+\alpha\psi)}\pi^L \end{aligned}$$

When credibility is perfect, $\lambda = 1$, then $x_t = 0$ and $\pi_t = \pi^L$. The disinflation is achieved at time T^* with no loss in output. The sacrifice ratio is zero.

When $\lambda < 1$, agents rely on past data to update their expectations and do not fully adjust to the new target upon implementation. Inflation expectations at time T^* are

$$\hat{\mathbb{E}}_t \pi_{t+1} = \lambda \pi^L + (1-\lambda)\pi^H.$$

The fact that inflation expectations remain somewhat anchored at the old target results in an insufficient decrease of inflation relative to the new inflation target. If the Taylor principle is satisfied, $\alpha > 1$, monetary policy generates a demand-driven recession by increasing the real interest rate to bring inflation towards its new lower target. The expressions above show that with $(\alpha - 1) > 0$ and $\pi^L < \pi^H$, the disinflation leads to a loss of output, $x_t < 0$. The loss of output is larger the more aggressive is the response of monetary policy to inflation because this determines the response of the real interest rate.[4] The loss of output decreases with credibility, consistent with the alternative specifications of imperfect credibility proposed in the disinflation literature.[5]

An anticipated cold-turkey disinflation works much the same way. The central bank announces the policy before it implements the new target. For this example let $T^a = T^* - 1$, where T^a is the time the announcement is made, and T^* now stands for the time of implementation. Because this corresponds to an anticipated policy change we must use the recursions given by Equations (12) to (14). Equation (13) implies $\mathbf{Q}_t = 0$ because $\mathbf{A} = 0$ and Equation (14) implies $\mathbf{G}_t = 0$ as $\mathbf{D} = 0$ in this case. Solving Equation (12), $\mathbf{C}_t = \mathbf{\Gamma} + \mathbf{B}\mathbf{C}_{t+1}$, implies:

$$\mathbf{C}_{T^a} = \mathbf{\Gamma} + \mathbf{B}\mathbf{C}^L,$$

and with imperfectly anchored expectations, $\hat{\mathbb{E}}_t y_{t+1} = \lambda \mathbf{C}_{T^a} + (1-\lambda)\mathbf{C}^H$.

The output gap and inflation at the time of the announcement are given by the expressions below:

$$\begin{aligned} x_{T^a} &= \lambda \frac{(\pi^H - \pi^L)(\alpha - 1)}{1 + \alpha\psi} \\ \pi_{T^a} &= \pi^H - \lambda \frac{(\pi^H - \pi^L)(1 + \psi)}{1 + \alpha\psi} \end{aligned}$$

The anticipation effect depends on credibility, λ. In the extreme case in which $\lambda = 0$, the expressions above reveal that the announcement has no impact. However, if $\lambda > 0$, the announcement influences output and inflation. The change in the inflation target and the aggressiveness of monetary policy matter. In particular, an anticipated disinflation results in a boom ($x_t > 0$) if the Taylor principle is satisfied ($\alpha > 1$). The intuition is that the news of an impending disinflation causes expectations of inflation and actual

[4] One can verify that $\partial x_t / \partial \alpha < 0$ provided $\psi > -1$, which is satisfied by model's theoretical restrictions. Interestingly, if monetary policy responds one-for-one to inflation, then $x_t = 0$ for all λ.

[5] If the Taylor principle is not satisfied ($\alpha < 1$), the initial impact leads to a boom in output. Therefore, on impact, passive monetary policy leads to lower output losses.

inflation to decrease. The fall in inflation represents a deviation from the current inflation target, π^H, so the central bank reduces the nominal interest rate. If the Taylor Principle is satisfied, $\alpha > 1$, the decrease in the nominal interest rate causes a decrease in the real interest rate and a corresponding boom in output. When the Taylor Principle is not satisfied, output falls.

Perhaps surprisingly, even with imperfect credibility, a pre-announcement can be beneficial. This is because, even with only a small amount of credibility, inflation still moves in the direction of the new inflation target. This movement then feeds back on itself through the adaptive learning component of expectations moving expectation closer to the new target faster, which lowers the sacrifice ratio.

One may wonder if increasing the time between the announcement and the implementation always increases the boom in output. In general, the answer is no. In fact, the farther apart the announcement and implementation dates, the smaller is the impact of the announcement on inflation and output. At a mechanical level, this is because the recursion is just a repeated iteration of the mapping from expectations to the structural parameters, and when the mapping is E-stable, beliefs converge to the old steady state and initial impacts vanish. At an intuitive level, it is because announcements regarding policy changes that will take place far into the future have little contemporaneous impact. But a boom would still take place as the implementation date approaches.

2.2.2 Convergence

Provided $\lambda < 1$, inflation expectations and inflation would not be at the new inflation target once the policy is implemented. Convergence to the new steady state hinges on adaptive learning. The adaptive learning rule that corresponds to the MSV solution in this case is a recursively estimated mean:

$$\mathbf{C}_t = \mathbf{C}_{t-1} + \gamma(y_{t-1} - \mathbf{C}_{t-1}). \tag{18}$$

Expectations are $\hat{\mathbb{E}}_t y_{t+1} = \lambda \mathbf{C} + (1-\lambda)\mathbf{C}_t$ and the actual law of motion is

$$y_t = \mathbf{\Gamma} + \mathbf{B}(\lambda \mathbf{C} + (1-\lambda)\mathbf{C}_t). \tag{19}$$

Substituting Equation (19) into Equation (18) results in

$$\mathbf{C}_t = \gamma\left(\mathbf{\Gamma} + \mathbf{B}\lambda\mathbf{C}\right) + \left(I + \gamma\left((1-\lambda)\mathbf{B} - I\right)\right)\mathbf{C}_{t-1}, \tag{20}$$

which is a stationary VAR process around $\mathbf{C^H}$ or $\mathbf{C^L}$ if

$$\alpha > \frac{\psi - \psi\lambda - \lambda^2}{\psi}.$$

If monetary policy is sufficiently aggressive, inflation will converge to $\mathbf{C}^L$ following implementation. Notice that setting $\lambda = 0$ recovers the familiar E-stability condition for this model that $\alpha > 1$.

Figure 1 shows the global convergence properties for an unanticipated disinflation under active and passive monetary policy. The two large dots on the figure correspond to $(0, \pi^H)$, the high inflation steady state, and to $(0, \pi^L)$, the low inflation steady state. The solid line describes the paths of output and inflation taking as initial condition the high inflation steady state. We compute paths for $\lambda = 0.1$ (left) or for $\lambda = 1/2$ (right). With almost no credibility, inflation can only be lowered through a demand contraction. The path to steady state is long and overshoots once in the neighborhood of the new target. With some credibility, though, inflation and output jump upon the simultaneous announcement and implementation of the policy. The path is more direct because more credibility pulls expectations toward the new steady state. In the case of full credibility, there are no convergence dynamics; beliefs jump from the high steady state to the low steady state without output moving.

Credibility has a significant effect on the dynamics of convergence. But the paths also depends on the monetary policy response. The larger α is, the more severe the counterclockwise swirl of Figure 1 and the higher the implied sacrifice ratio. As α decreases, the swirl diminishes. If the Taylor principle is not satisfied, $\alpha < 1$, as shown in the bottom two panels, the swirl becomes clockwise. Note that convergence when the $\alpha < 1$ is because of the anchoring of expectations. The weight placed on the rational belief counteracts the adaptive belief and pushes expectations towards steady state. Of course, if credibility is too low in this case, then inflation and inflation expectations would not converge and both would explode away from steady state.

Finally, Figure 1 also provide insight on how shocks affect the cost of disinflation and the time to disinflate. When credibility is low, a shock that reduces inflation lowers the sacrifice ratio. Shocks that increase inflation unambiguously place the economy on trajectories that imply longer times to converge and larger output losses. As credibility increases, the cost of bad shocks is reduced because paths to the low inflation steady state are more direct. The next section makes this point explicit in the context of a model with many shocks.

2.3 Comparison to imperfect credibility in the literature

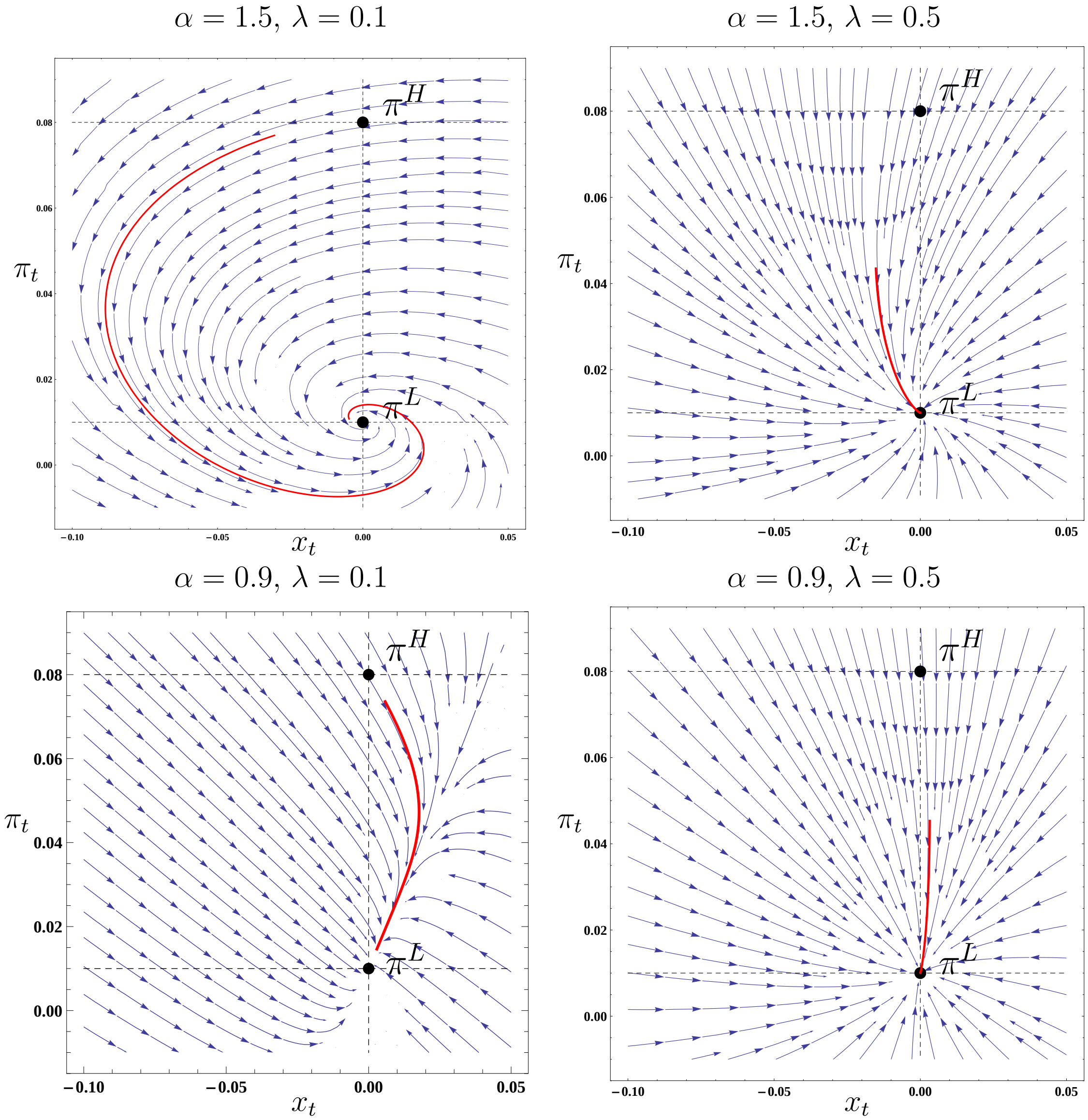
The most common way imperfect credibility is modeled in the literature is as agents placing some weight in their expectations on the central bank returning to the old inflation target. For example, Ascari and Ropele (2013) models imperfectly credibility in medium scale DSGE model by assuming that agents' expectations take the following form

$$\hat{\mathbb{E}}_t \pi_{t+1} = (1 - \lambda_t)\mathbb{E}_t \pi_{t+1} + \lambda_t \bar{\pi}^{old}, \tag{21}$$

where $\lambda_t = \rho \lambda_{t-1}$ is a deterministic AR(1) process and $\mathbb{E}_t$ is the standard rational expectations operator. The assumption of an AR(1) process implies that the policy becomes gradually more credible over time. This assumption introduces a wedge between expectations of inflation and actual inflation causing the real interest rate to rise when the inflation target is lowered, which increases the cost of a disinflation.

The principle difference between this assumption for modeling credibility and the one we consider, imperfectly anchored expectations, is that beliefs do not depend on the evolution of data. Placing weight on the old inflation target increases the mean sacrifice ratio but it does not propagate shocks. In our framework, when credibility is

Figure 1: Global convergence



Notes: Illustration of convergence to new steady state following the implementation of a disinflation. The solid line indicates the non-stochastic path assuming inflation and output gap are at steady state upon implementation of the new policy.

low, adaptive learning does propagate shocks, which increases both the average costs and the variance of the cost for a given disinflation policy.

To illustrate, Figure 2 compares an imperfectly credible anticipated disinflation using the two frameworks. The disinflation is announced in period 3 and the policy is put into place in period 5. The λ parameter in both cases is initially set to 0.5. It remains fixed at that level in our framework and decays at the rate $\rho = 0.05$ in the framework of Ascari and Ropelle.[6] Because the policy is anticipated, there is initially a boom in output as the real interest rate falls. However, once the policy is put in place, the real interest rate rises and output contracts. The contraction is short lived in the first case as weight on old inflation target quickly dissipates under this parameterization, while both the fall in inflation and output losses are persistence in the other case because of adaptive learning. The sacrifice ratio for the two scenarios, however, are almost identical at -0.03 and -0.029, respectively.

Figure 3 shows the difference between the two cases when shocks are considered. In the first column, we show an unanticipated positive demand shock for the same disinflation considered in the previous figure. The shock are of equal size in both cases but have dramatically different effects. Under the Ascari and Ropelle assumption, the shock increases the duration of the disinflation by one quarter. But, because the shock positively impacts output, the sacrifice ratio decreases to -0.043. In our framework, the initial impact of the shock is the same. But, because expectations depend on data, the shock has a lasting effect. The duration of the disinflation increases by about 4 quarters and the sacrifice ratio increases to 0.08.[7]

In the second column of Figure 3, we show the impact of a negative demand shock. This causes the disinflation to end one quarter earlier in the Ascari and Ropelle case but leaves the sacrifice ratio largely unchanged (-0.27). In our framework, the shock also causes the disinflation to end about a quarter earlier than it otherwise would have and lowers the sacrifice ratio to -0.038. This illustrates the asymmetric effects that our framework can generate. Bad luck shocks, like the positive demand shock, can significantly impact the cost and duration of the disinflation because data becomes inconsistent with the policy objectives of the central bank. While good luck shocks like the negative demand shock, push inflation in the direction of the central bank's policy, which ends the disinflation much sooner than may otherwise occur.

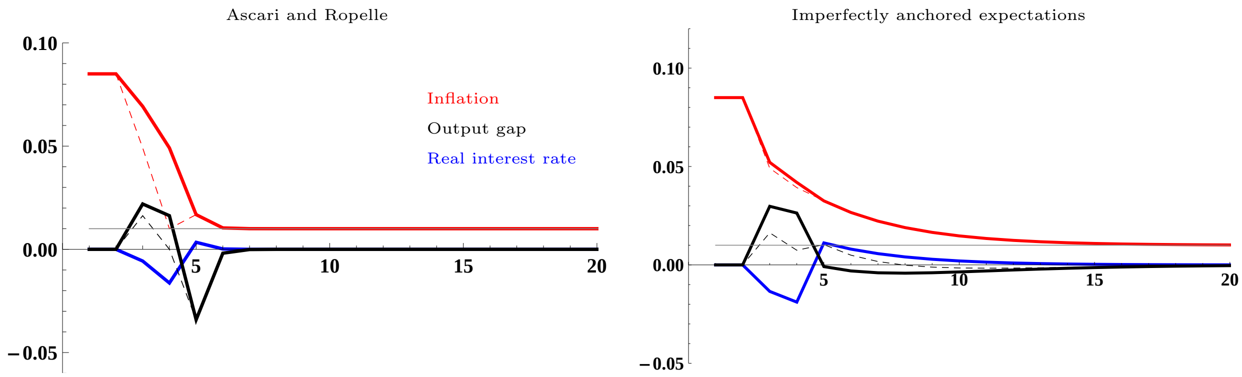
3 Opportunism and the size of shocks

The key to an opportunistic disinflation strategy is that the public sector associates the movements in inflation that occur by chance with the policy announcements of the central bank. Inflation is of course always moving around the inflation target over time with the size of these movements depending on a host of different factors in the economy. Therefore, the effectiveness of this policy hinges on how large a movement in inflation has occurred relative to the recent inflation volatility, when the central bank announces

[6] The remaining parameters are $\alpha = 1.5$, $\psi = 0.1$, $\gamma = 0.5$.

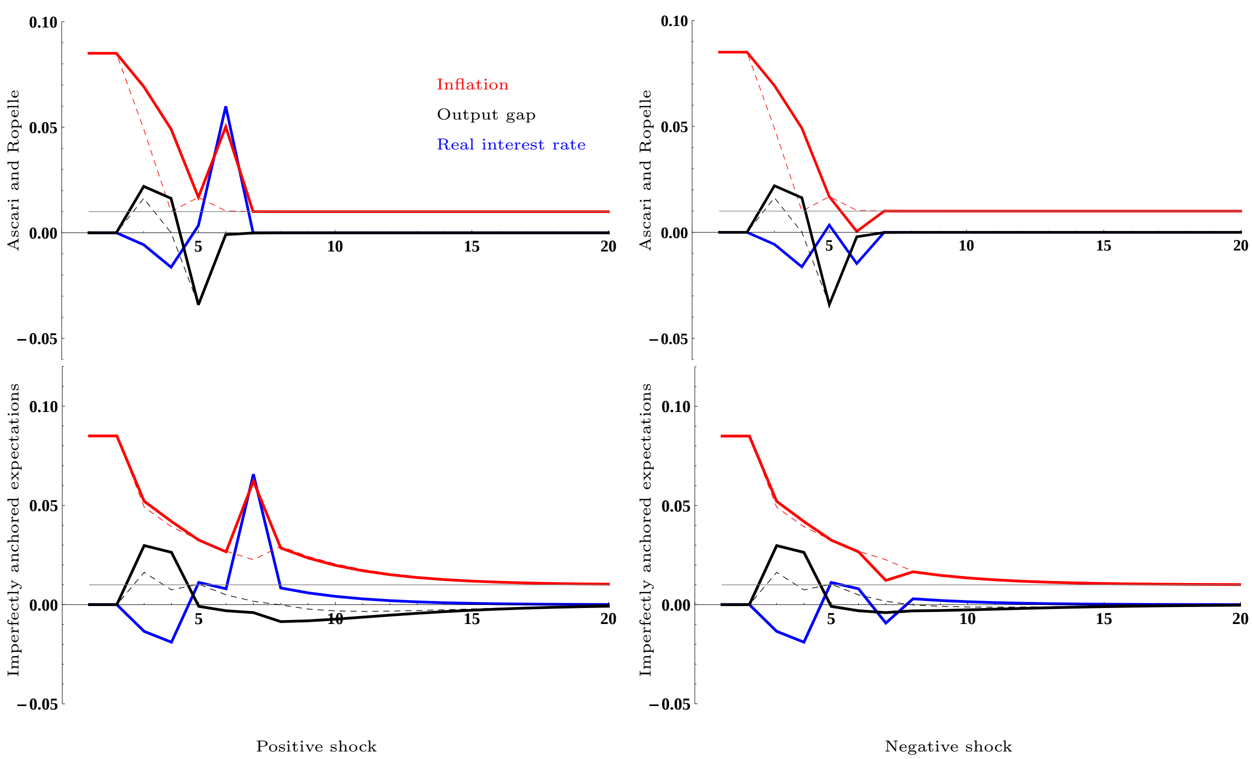
[7] Changing the ρ parameter in Ascari and Ropelle specification increases the cost and duration of the disinflation but does not affect the model ability to propagate shocks.

Figure 2: Comparison of two specifications of credibility



Notes: Disinflation example using the specification of Ascari and Ropele (2013) (left) and imperfectly anchored expectations (right). The disinflation is announced in time period 3 and implemented in time period 5. Solid lines show the path of the variables and the dashed lines show the expectation of the variable.

Figure 3: Comparison of two specifications of credibility with one-time shocks



Notes: Disinflation example using the specification of Ascari and Ropele (2013) (left) and imperfectly anchored expectations (right). The disinflation is announced in time period 3 and implemented in time period 5. Solid lines show the path of the variables and the dashed lines show the expectation of the variable.

Table 1: Policy comparisons

	$\lambda = 0.33$	$\lambda = 0.66$	$\lambda = 1$
Argentina			
Mean:	3.82	1.68	0.91
St. Dev:	7.38	5.58	6.99
Skewness:	2.77	1.05	-0.93
Kurtosis:	17.53	15.86	8.50
Argentina with US Shocks			
Mean:	3.03	1.63	1.69
St. Dev:	1.76	0.89	0.82
Skewness:	1.72	0.92	0.10
Kurtosis:	7.45	3.75	3.09

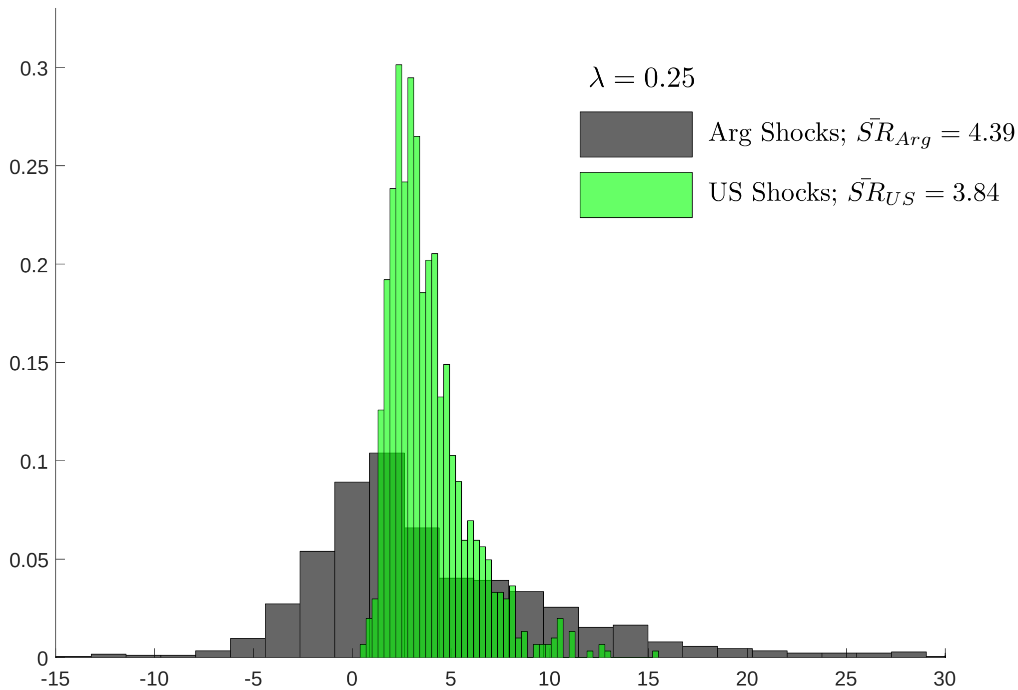
Notes: This table compares the effect of the size of shocks for the distribution of the sacrifice ratio for different levels of credibility.

the new policy. In terms of the model, the speed at which agents update their beliefs is relative to the variance of the shocks in the economy. For example, small changes in inflation when inflation volatility has been high lead to very small revisions in inflation expectations.

Table 1 illustrate this effect by comparing the distribution of sacrifice ratios for 30% disinflation implied by the Argentina calibration of the model to the same calibration but with the shocks set to the smaller US values. The results show that the size of the shocks matter greatly when credibility is low. Large shocks increase the probability of a very costly disinflation, while simultaneously increasing the possibility of a costless disinflation. Figure 4 further illustrates this pattern by showing the distribution for a case in which there is low credibility ($\lambda = 0.25$). Both tails of the distribution increase with large shocks.

The mechanism that drives this divergence under the two different shock regimes is the interaction of the size of the shocks with adaptive learning. Large shocks increase the chance of large movements in inflation towards and away from target. This good or bad luck drives the outcomes seen at both tails of the distributions. This natural variation in inflaiton and its effect on expectations is exploitable by a central bank to engineer less costly disinflations.

Figure 4: The size of shocks and the distribution of the sacrifice ratio



Notes: Distribution of the sacrifice ratio for 30% disinflation under the Argentinean calibrations for the policy and structural parameters. The shock parameters are set to the Argentinean values for the first distribution and the US values to construct the second distribution. $\bar{SR}_{Arg}$ is the mean sacrifice ratio of with large shocks and $\bar{SR}_{US}$ is the mean sacrifice ratio with small shocks.

References

Ascari, Guido and Tiziano Ropele, "Disinflation effects in a medium-scale new keynesian model: money supply rule versus interest rate rule," *European Economic Review*, 2013, *61*, 77–100.

Ball, Laurence, "Credible disinflation with staggered price-setting," *The American Economic Review*, 1994, pp. 282–289.

Binder, Michael and M Hashem Pesaran, "Multivariate Rational Expectations Models and Macroeconometric Modelling: A Review and Some New Results," in in M Hashem Pesaran and M Wickens, eds., *Handbook of Applied Econometrics: Macroeconomics*, Basil Blackwell Oxford 1995, pp. 139–187.

Cagliarini, Adam and Mariano Kulish, "Solving Linear Rational Expectations Models with Predictable Structural Changes," *Review of Economics and Statistics*, 2013, *95*, 328–336.

Evans, George W. and Seppo Honkapohja, *Learning and Expectations in Macroeconomics*, Princeton: Princeton University Press, 2001.

Kulish, Mariano and Adrian Pagan, "Estimation and solution of models with expectations and structural changes," *Journal of Applied Econometrics*, 2017.

McCallum, Bennett T, "On non-uniqueness in rational expectations models: An attempt at perspective," *Journal of monetary Economics*, 1983, *11* (2), 139–168.