Online Appendix

The sacrifice ratio and active fiscal policy

Christopher G. Gibbs* Herbert W. Xin†

June 17, 2024

1 Simple Model

We solve the simple model using the method of undetermined coefficients. We write the model as

$$\begin{pmatrix} x_t \\ \pi_t \\ b_t \end{pmatrix} = \underbrace{\begin{pmatrix} \frac{\bar{\pi}\sigma(\beta\phi_\pi - 1)}{\kappa\sigma\phi_\pi + 1} \\ \frac{\bar{\pi}(-\beta + \kappa\sigma(\phi_\pi - 1) + 1)}{\kappa\sigma\phi + 1} \\ \frac{\bar{\pi}(-\phi_s + \phi_\pi - 1)(\beta + \kappa\sigma)}{\beta\kappa\sigma\phi_\pi + \beta} \end{pmatrix}}_{A}$$

$$+ \underbrace{\begin{pmatrix} \frac{1}{\kappa\sigma\phi_\pi + 1} & \frac{\sigma - \beta\sigma\phi_\pi}{\kappa\sigma\phi + 1} & 0 \\ \frac{\kappa}{\kappa\sigma\phi_\pi + 1} & \frac{\beta + \kappa\sigma}{\kappa\sigma\phi_\pi + 1} & 0 \\ \frac{\kappa(\phi_s - \phi_\pi + 1)}{\beta\kappa\sigma\phi_\pi + \beta} & -\frac{(-\phi_s + \phi_\pi - 1)(\beta + \kappa\sigma)}{\beta\kappa\sigma\phi_\pi + \beta} & 0 \end{pmatrix}}_{B} \begin{pmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \\ E_t b_{t+1} \end{pmatrix} + \underbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1-\delta}{\beta} \end{pmatrix}}_{C} \begin{pmatrix} x_{t-1} \\ \pi_{t-1} \\ b_{t-1} \end{pmatrix}.$$

The equilibria are the solutions the following system of matrix equations

$$\begin{aligned} a &= (I - cB)^{-1} (A + Ba) \\ c &= (I - cB)^{-1} C \end{aligned}$$

*University of Sydney. E-mail: christopher.gibbs@sydney.edu.au
†University of Oregon. E-mail: hxin@uoregon.edu.

There are three solutions

$$\bar{a} = \begin{pmatrix} 0 \\ \bar{\pi} \\ 0 \end{pmatrix}, \bar{c}_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1-\delta}{\beta} \end{pmatrix},$$

$$\bar{c}_2 = \begin{pmatrix} 0 & 0 & \frac{\left(\sqrt{\beta^2+\beta(\kappa\sigma(2-4\phi_\pi)-2)+(\kappa\sigma+1)^2}+\beta+\kappa\sigma-1\right)\left(\sqrt{\beta^2+\beta(\kappa\sigma(2-4\phi_\pi)-2)+(\kappa\sigma+1)^2}+\beta+2\delta+\kappa\sigma-1\right)}{4\kappa(-\phi_s+\phi_\pi-1)} \\ 0 & 0 & -\frac{\sqrt{\beta^2+\beta(\kappa\sigma(2-4\phi_\pi)-2)+(\kappa\sigma+1)^2}+\beta+2\delta+\kappa\sigma-1}{2(-\phi_s+\phi_\pi-1)} \\ 0 & 0 & \frac{\sqrt{\beta^2+\beta(\kappa\sigma(2-4\phi_\pi)-2)+(\kappa\sigma+1)^2}+\beta+\kappa\sigma+1}{2\beta} \end{pmatrix},$$

and

$$\bar{c}_3 = \begin{pmatrix} 0 & 0 & \frac{\left(-\sqrt{\beta^2+\beta(\kappa\sigma(2-4\phi_\pi)-2)+(\kappa\sigma+1)^2}+\beta+\kappa\sigma-1\right)\left(-\sqrt{\beta^2+\beta(\kappa\sigma(2-4\phi_\pi)-2)+(\kappa\sigma+1)^2}+\beta+2\delta+\kappa\sigma-1\right)}{4\kappa(-\phi_s+\phi_\pi-1)} \\ 0 & 0 & -\frac{-\sqrt{\beta^2+\beta(\kappa\sigma(2-4\phi_\pi)-2)+(\kappa\sigma+1)^2}+\beta+2\delta+\kappa\sigma-1}{2(-\phi_s+\phi_\pi-1)} \\ 0 & 0 & \frac{-\sqrt{\beta^2+\beta(\kappa\sigma(2-4\phi_\pi)-2)+(\kappa\sigma+1)^2}+\beta+\kappa\sigma+1}{2\beta} \end{pmatrix}.$$

Only $\bar{c}_1$ and $\bar{c}_3$ imply stationary process for debt. Which equilibrium selected depends on ϕ_π and δ. When $\phi_\pi > 1$ and $1-\beta < \delta < 1+\beta$, $\bar{c}_1$ is the unique stationary equilibrium. When $\phi_\pi < 1$ and $0 \leq \delta < 1-\beta$, then $\bar{c}_3$ is the unique stationary equilibrium.

We make the claim that equilibrium selection does not depend on ϕ_s in Section 2. You can see that this claim is true by noting that the entry in the third column and third row of $\bar{c}_1$, $\bar{c}_2$, and $\bar{c}_3$ are not functions of ϕ_s. The value of this parameter does not effect which of the equilibrium are stationary and hence which equilibrium ϕ_π and δ select.

We can also see the irrelevance of ϕ_s by checking the eigenvalues that govern determinacy. Advance the debt equation one period in time such that

$$\beta^{-1}(1-\delta)b_t = E_t b_{t+1} + \beta^{-1}(\phi_\pi - 1 - \phi_s)E_t\pi_{t+1}$$

We can then write the matrix in front of expectations as

$$\tilde{B} = \begin{pmatrix} \frac{1}{\kappa\sigma\phi_\pi+1} & \frac{\sigma-\beta\sigma\phi_\pi}{\kappa\sigma\phi_\pi+1} & 0 \\ \frac{\kappa}{\kappa\sigma\phi_\pi+1} & \frac{\beta+\kappa\sigma}{\kappa\sigma\phi_\pi+1} & 0 \\ 0 & \frac{\phi_s-\phi_\pi+1}{\delta-1} & -\frac{\beta}{\delta-1} \end{pmatrix}$$

The requirement for determinacy of the model is that two of the three eigenvalues $\tilde{B}$ be inside the unit circle. The relevant eigenvalues are

$$\lambda_1 = \frac{\beta}{1-\delta}$$

and

$$\lambda_1, \lambda_2 = \frac{\beta\delta - \beta + \delta\kappa\sigma + \delta - \kappa\sigma - 1 \pm (\delta - 1)\sqrt{\beta^2 - 4\beta\kappa\sigma\phi_\pi + 2\beta\kappa\sigma - 2\beta + \kappa^2\sigma^2 + 2\kappa\sigma + 1}}{2(\delta - 1)(\kappa\sigma\phi_\pi + 1)}.$$

The parameter ϕ_s does not affect determinacy.

2 Wage Stickness

In a standard New Keynesian framework with Rotemberg price adjustment costs as the sole friction, we find that linking fiscal policy to inflation does not reduce the cost of disinflation. In fact, such a policy linkage is detrimental. However, this result is overturned in the model of Smets and Wouters (2007), where a stronger fiscal policy response to changes in inflation can reduce the sacrifice ratio.

In particular, we demonstrate that the presence of wage rigidity is the key factor behind this reversal in the relationship between fiscal-inflation policy coordination and sacrifice ratio. This finding is robust across both the simple New Keynesian setup and the more comprehensive Smets-Wouters framework.

Table 1 reports the sacrifice ratios from simple New Keynesian model with a purely forward-looking Philips curve, the model described in the paper, and a hybrid Philips curve with inflation indexation. We found the introduction of wage stickiness in combination with long-term debt results changes the relationship between fiscal policy reaction and inflation, as tying fiscal policy with inflation starts to produce lower sacrifice ratios.

In the Smets-Wouters model, which features two extra nominal rigidities, captial adjustment cost and wage rigidity, we find that capital adjustment costs do not play a role, but the degree of wage stickiness is crucial. Table 2 reports the sacrifice ratios with different capital adjustment costs, the upper panel outlines the sacrifice ratios with short-term debt and the lower panel

Table 1: Sacrifice Ratios - Simple New Keynesian Model

6% Disinflation	**Monetary**		**Fiscal**					
	w/ peg	*w/o peg*	*w/ peg*			*w/o peg*		
	-	$\phi_\pi = 1.5$	-	-	-	$\phi_\pi = 0.5$	$\phi_\pi = 0.5$	$\phi_\pi = 0.5$
	$\phi_s = 0.5$	$\phi_s = 0.5$	$\phi_s = 0.0$	$\phi_s = 0.5$	$\phi_s = 1.5$	$\phi_s = 0.0$	$\phi_s = 0.5$	$\phi_s = 1.5$
w/ short-term debt								
Forward-looking								
Cold-turkey	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Announced (j = 1)	0.25	-0.11	**-0.64**	-0.48	-0.38	-0.37	-0.37	-0.37
Announced (j = 2)	0.77	-0.30	**-1.03**	-0.78	-0.63	-0.65	-0.65	-0.65
Hybrid								
Cold-turkey	0.5739	0.5739	0.5479	0.5479	0.5479	0.5479	0.5479	0.5479
Announced (j = 1)	0.6119	0.5124	**0.4127**	0.4417	0.4577	0.4480	0.4480	0.4480
Announced (j = 2)	0.8139	0.4253	**0.3350**	0.3957	0.4336	0.3911	0.3911	0.3911
w/ long-term debt								
Forward-looking								
Cold-turkey	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Announced (j = 1)	0.25	-0.11	**-0.43**	-0.37	-0.33	**-0.47**	-0.43	-0.40
Announced (j = 2)	0.77	-0.30	**-0.75**	-0.63	-0.55	**-0.84**	-0.76	-0.71
Hybrid								
Cold-turkey	0.5739	0.5739	0.5884	0.5728	**0.5620**	0.5884	0.5728	**0.5620**
Announced (j = 1)	0.6119	0.5124	0.4755	0.4752	**0.4750**	0.4564	0.4532	**0.4510**
Announced (j = 2)	0.8139	0.4253	**0.4280**	0.4468	0.4603	**0.3755**	0.3815	0.3857

Notes: Sacrifice ratios for different disinflation policies. The lowest sacrifice ratio within a regime set is bolded. The shared parameters are $\beta = 0.995$, $\sigma = 1$, $\kappa = 0.1$. Under the monetary led regime, we set $R = 0.35$. Under the fiscal led regime, we set $R = 0.0$. For the long-debt specification, we set $\rho = 0.85$.

Table 2: Sacrifice Ratios - Smets and Wouters (2007) Model

SW(2007)	**Monetary**		**Fiscal**					
	w/ peg	*w/o peg*	*w/ peg*			*w/o peg*		
			$\phi_s = 0$	$\phi_s = 0.5$	$\phi_s = 1.5$	$\phi_s = 0$	$\phi_s = 0.5$	$\phi_s = 1.5$
w/ short-term debt								
Cold-turkey								
adjustcost = 1	0.6844	0.6844	1.5510	0.8579	0.7006	1.5510	0.8579	0.7006
adjustcost = 3	0.5794	0.5794	1.2612	0.7679	0.6418	1.2612	0.7679	0.6418
adjustcost = 5	0.5201	0.5201	1.1077	0.7094	0.6008	1.1077	0.7094	0.6008
Announced (j = 1)								
adjustcost = 1	2.3552	-0.4713	0.9882	0.3579	0.2728	-1.4458	-0.7985	-0.6515
adjustcost = 3	2.4770	-0.3520	1.5519	0.6571	0.5105	-1.1036	-0.6711	-0.5605
adjustcost = 5	2.3947	-0.2908	1.7021	0.7999	0.6324	-0.9217	-0.5896	-0.4991
Announced (j = 2)								
adjustcost = 1	2.7019	-1.5102	-1.2482	-0.2767	-0.1952	-3.6961	-2.0431	-1.6678
adjustcost = 3	3.1591	-1.2445	1.0109	0.2793	0.2074	-3.0592	-1.8615	-1.5552
adjustcost = 5	3.1626	-1.0809	2.0941	0.6681	0.4998	-2.6474	-1.6944	-1.4346
w/ long-term debt								
Cold-turkey								
adjustcost = 1	0.6844	0.6844	0.9965	0.7764	0.6802	0.9965	0.7764	0.6802
adjustcost = 3	0.5794	0.5794	0.9093	0.7153	0.6299	0.9093	0.7153	0.6299
adjustcost = 5	0.5201	0.5201	0.8488	0.6720	0.5936	0.8488	0.6720	0.5936
Announced (j = 1)								
adjustcost = 1	2.3552	-0.4713	1.2712	0.6005	0.3876	-0.9197	-0.7186	-0.6307
adjustcost = 3	2.4770	-0.3520	1.4318	0.7966	0.5859	-0.8027	-0.6284	-0.5517
adjustcost = 5	2.3947	-0.2908	1.5067	0.8997	0.6916	-0.7250	-0.5673	-0.4973
Announced (j = 2)								
adjustcost = 1	2.7019	-1.5102	0.9610	0.1873	-0.0085	-2.4981	-1.9054	-1.6463
adjustcost = 3	3.1591	-1.2445	1.5230	0.5833	0.3409	-2.2816	-1.7689	-1.5431
adjustcost = 5	3.1626	-1.0809	1.9280	0.8847	0.6048	-2.1040	-1.6400	-1.4342

*N*otes: *adjustcost* stands for the capital adjustment cost. Larger *adjustcost* means higher capital adjustment cost

report those with long-term debt. Results from Table 2 suggest that it is beneficial by tying fiscal policy with inflation in both scenarios, preserving the relationship in the baseline Smets and Wouters (2007) model. The relationship flips when wage rigidity changes. When wage rigidity is low, tying fiscal policy to inflation remains detrimental, while when wage rigidity is high, a stronger fiscal policy response to inflation produces better macroeconomic outcomes, as outlined by Table 3.

In conclusion, we found wage stickiness caused the results to change when going from the simple model to the medium-scale model.

Table 3: Sacrifice Ratios - Smets and Wouters (2007) Model

6% Disinflation	**Monetary**		**Fiscal**					
	w/ peg	*w/o peg*	*w/ peg*			*w/o peg*		
	$\phi_\pi = 1.5$	-	$\phi_\pi = 0.5$	$\phi_\pi = 0.5$	$\phi_\pi = 0.5$	-	-	-
	$\phi_s = 0.5$	$\phi_s = 0.5$	$\phi_s = 0.0$	$\phi_s = 0.5$	$\phi_s = 1.5$	$\phi_s = 0.0$	$\phi_s = 0.5$	$\phi_s = 1.5$
w/ short-term debt								
wagecal = 0.7	0.5201	0.5201	1.1077	0.7094	**0.6008**	1.1077	0.7094	0.6008
wagecal = 3	0.2764	0.2764	0.5465	0.3964	**0.3483**	0.5465	0.3964	0.3483
wagecal = 5	0.2584	0.2584	0.5038	0.3692	**0.3254**	0.5038	0.3692	0.3254
Announced (j = 1)								
wagecal = 0.7	2.3947	-0.2908	1.7021	0.7999	**0.6324**	-0.9217	-0.5896	-0.4991
wagecal = 3	0.1040	-0.3684	**-0.4315**	-0.2950	-0.2544	-0.7949	-0.5766	-0.5066
wagecal = 5	-0.1568	-0.3726	**-0.6137**	-0.4371	-0.3818	-0.7824	-0.5734	-0.5054
Announced (j = 2)								
wagecal = 0.7	3.1626	-1.0809	2.0941	0.6681	**0.4998**	-2.6474	-1.6944	-1.4346
wagecal = 3	0.1684	-0.9827	**-1.1050**	-0.6649	-0.5538	-1.9204	-1.3932	-1.2242
wagecal = 5	-0.2583	-0.9710	**-1.3898**	-0.9099	-0.7752	-1.8589	-1.3623	-1.2010
w/ long-term debt								
Cold-turkey								
wagecal = 0.7	0.5201	0.5201	0.8488	0.6720	**0.5936**	0.8488	0.6720	0.5936
wagecal = 3	0.2764	0.2764	0.4195	0.3647	**0.3375**	0.4195	0.3647	0.3375
wagecal = 5	0.2584	0.2584	0.3855	0.3382	**0.3145**	0.3855	0.3382	0.3145
Announced (j = 1)								
wagecal = 0.7	2.3947	-0.2908	1.5067	0.8997	**0.6916**	-0.7250	-0.5673	-0.4973
wagecal = 3	0.1040	-0.3684	**-0.2888**	-0.2545	-0.2386	-0.6751	-0.5627	-0.5070
wagecal = 5	-0.1568	-0.3726	**-0.4627**	-0.3989	-0.3684	-0.6698	-0.5608	-0.5062
Announced (j = 2)								
wagecal = 0.7	3.1626	-1.0809	1.9280	0.8847	**0.6048**	-2.1040	-1.6400	-1.4342
wagecal = 3	0.1684	-0.9827	**-0.7070**	-0.5770	-0.5227	-1.6203	-1.3542	-1.2224
wagecal = 5	-0.2583	-0.9710	**-1.0062**	-0.8274	-0.7482	-1.5797	-1.3266	-1.200

*N*otes: *wagecal* stands for the Calvo parameter of wage. Larger *wagecal* means less wage stickiness.

3 Calibration

Parameter	Value	Description
σ	1	Intertemporal elasticity of substitution
β	0.995	Time discount factor
κ	0.1	Output-Inflation sensitivity
ϕ_{i}	1.5 (0.5)	Monetary policy (Under fiscal-led regime)
ϕ_{s}	0, 0.5, 1.5	Surplus responsiveness to inflation
ρ	0.85	Maturity structure of long-term debt
α_1	0.5	Inflation indexation
α_2	0.5	Inflation forward-lookingness
δ	0.5 (0)	Fiscal policy (Under fiscal-led regime)
γ	0.5	Autoregressive debt parameter

Table 4: Parameter calibration for simple New Keynesian model

Table 5: Parameter calibration for Smets-Wouters model

Parameters	Value	Parameters	Value
Structural parameters			
ϕ	5.7606	ψ	0.5462
σ_c	1.3808	Φ	1.6064
h	0.7133	r_π	2.0443 (0.5)
ξ_w	0.7061	ρ	0.8103
σ_l	1.8383	r_y	0.0882
ξ_p	0.6523	$r_{\Delta y}$	0.2247
ι_w	0.5845	α	0.24
ι_p	0.2432	δ	0 (2.0443)
ρ_{maturity}	0.85	ϕ_s	0.2751
γ	0.5	β_v	0.9995
Autoregressive parameters			
ρ_a	0.9577	ρ_p	0.8895
ρ_b	0.2194	ρ_w	0.9688
ρ_g	0.9767	μ_p	0.7010
ρ_I	0.7113	μ_w	0.8503
ρ_r	0.1479		

References

Smets, Frank and Rafael Wouters, "Shocks and frictions in US business cycles: A Bayesian DSGE approach," *American Economic Review*, 2007, *97* (3), 586–606.