A Unified Theory of Learning to Forecast

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Introduction and Motivation

- What we do
  1. Propose a **model of bounded rationality** that is based on experimental studies of strategic behavior in microeconomic and game theoretic settings
     - Allows for boundedly rational **forward-looking behaviour**
  2. Characterize its predictions in simple macroeconomic environments and compare to experimental evidence

Introduction and Motivation

- Why we do it

  1. There is now a large experimental literature to draw upon
  2. There is significant interest in **central bank announcements** about future policy

     - rational expectations (RE) make some **implausible/counter intuitive predictions**
     - behavioral rules and adaptive learning for the most part do not offer a way to incorporate such information

Summary of Method and Results

- We propose a model of boundedly rational expectation formation that combines elements of

    1. Adaptive learning (Evans and Honkapohja 2001)
    2. Behavioral heterogeneity (Brock and Hommes 1997; Hommes 2013; Branch and McGough 2008)
    3. Eductive learning (Guesnerie 1992 and 2002)
    4. Level-k reasoning (Nagel 1995)

Summary of Method and Results

- Summary of results:

  1. Show that eductive stability may be obtained through an adaptive process
  2. Show the proposed mechanism has a number of desirable properties
  3. Show that we can rationalize previously unexplained behavior in laboratory experiments
  4. Show that the model has some interesting testable implications for laboratory experiments.

Background on key concepts

Background

- Three bounded rationality approaches:
  - Adaptive Learning: people are about as smart as econometricians and employ simple econometric models estimated on past data to form expectations
    - Bray (1982), Marcet and Sargent (1989) and Evans and Honkapohja (2001)
  - Level-k reasoning: people have knowledge of the structure of the economy and believe that others are not rational. They best respond to what they believe other, less sophisticated players, will do
    - Nagel (1995) and Camerer et al (2004), Stahl and Wilson (1994 and 1995), Duffy and Nagel (1997), Ho et al (1998), Costa-Gomes and Crawford (2006)
  - Eductive Learning: people have knowledge of the structure of the economy and *common knowledge of rationality*
    - Guesnerie (1992) and Guesnerie (2002)

Background

Example: Nagel (1995) guess-the-average/beauty contest game

- The Game
  1. Ask a large number of players to simultaneously choose a number from 0 to 100
  2. The person who picks the number closest to $p\times$average wins
  3. Repeat the game with the same players multiple times

- The findings
  1. Nash prediction of zero is not observed
  2. People are heterogeneous and display finite depths of reasoning
  3. Evidence of some learning behavior both for level-0 agents and over depths of reasoning

The Model

The Model

$$y_t = \gamma + \beta \hat{E}_{t-1} y_t \tag{1}$$

$$y_t = \gamma + \beta \sum_{k=0}^{K} \omega_k E_{t-1}^k y_t, \text{ where } 0 \leq \omega_k \leq 1 \text{ and } \sum_{k=0}^{K} \omega_k = 1. \tag{2}$$

- $\beta < 1$ and we assume $\beta \neq 0, 1$
- Nests guess-the-average and the cobweb model
- K refers to the type of agent
- ω_k is the proportion of agents of type K

The Model

- The K-types
  - $k = 0$ or level 0 agents are adaptive learners such that $E^0_{t-1} y_t = a_{t-1}$

$$a_t = a_{t-1} + \phi(y_t - a_{t-1}) \tag{3}$$

  - $k = 1$ or level 1 agents believe all other market participants are level 0

$$E^1_{t-1} y_t = T(a), \text{ where } T(a) = \gamma + \beta a \tag{4}$$

  - $k = 2, 3, ..., K$ or level 2, 3, ..., K agents believe all other market participants are level $k - 1$

$$E^k_{t-1} y_t = T^k(a) \text{ for } k = 2, \ldots, K \tag{5}$$

  - $k = \infty$ or level ∞ agents are eductive learners

The Model with an Evolutionary Dynamic

- Giving agents a choice over k

  1. Agents are relatively inattentive to revise their depth of reasoning

     - Supported by empirical findings in Nagel (1995) and Nagel and Duffy (1997)

  2. Some may choose a rule that is not currently in use as long as it superior to all other rules

The Model with an Evolutionary Dynamic

Following Branch and McGough (2008), the proportion $\omega_{i,t}$ evolve as

$$\omega_{i,t} = \begin{cases} \omega_{i,t-1} - q_i(\omega_{t-1}) \sum_{j \in B_t} r(\bar{l}_t - l_{j,t})\omega_{j,t-1} & \text{if } i \in G_t \\ \\ (1 + r(\bar{l}_t - l_{i,t}))\omega_{i,t-1} & \text{if } i \in B_t \end{cases} \tag{6}$$

$$q_i(\omega_{t-1}) = \frac{\zeta/|G_t| + \bar{l}_t - l_{i,t}}{\zeta + \sum_{j \in G_t} \bar{l}_t - l_{j,t}}, \tag{7}$$

where r is a rate function

$$r : (-\infty, 0) \to (-1, \delta)$$

with $\delta \leq 0$ and $r' > 0$,

$$l_{i,t} = (y_t - E^i_{t-1} y_t)^2, \text{ and } \bar{l}_t = \sum_{i \in U_{t-1}} \omega_{i,t-1} l_{i,t}$$

Summary of theoretical results

Theoretical Results

1. Convergence to the RE steady state $\Rightarrow$ the familiar E-stability $\beta < 1$ condition and a modified eductive stability condition if level-∞ agents are considered: $|\beta| < 1/\omega_\infty$

2. Given $\beta < 1$ and uniform proportions $\{\omega_i\}_{i=j}^{N}$, where N is finite. The optimal level k is never ∞.

3. Under the evolutionary dynamic, $-1 < \beta < 1$, and in the absence of adaptive learning, the price $y_t \to \bar{y}$ and the depth of reasoning converges to infinity.

Dynamic predictions and experimental evidence

Experimental Evidence

Bao and Duffy EER 2016:

- Tests whether laboratory participants are adaptive learners or eductive learners in a traditional Cobweb model with negative feedback, where laboratory participants **know the structure of the market**
- Findings:
  - Evidence of both adaptive and eductive learning
  - Converge is faster than adaptive learning predicts but slower than eductive learning
  - **find bounded but non-covergent dynamics** when E-stability is satisfied but eductive stability is not i.e. **when** $\beta < -1$

Experimental Evidence

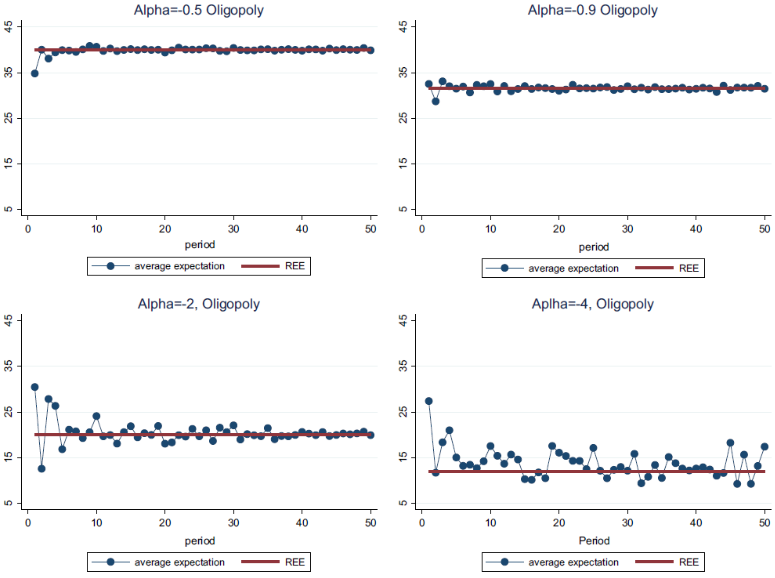


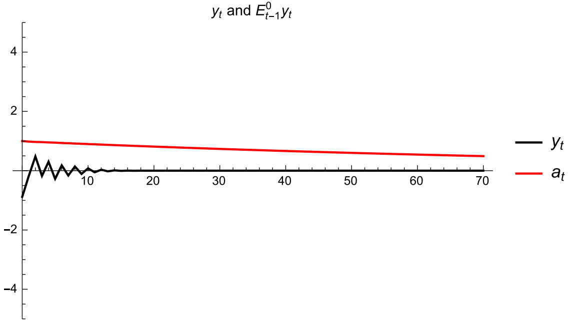
Fig. 5. The average expectation against the REE price when $\alpha = -0.5, -0.9, -2$ and -4 (from top to bottom) in the oligopoly design.

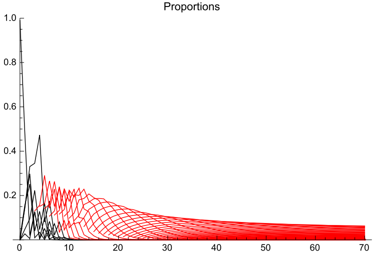
Dynamic predictions

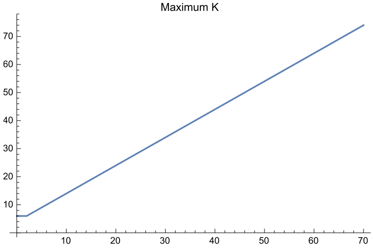
- Let's look at two examples:

  1. $\beta = -0.9$ and $\phi = 0.01$
  2. $\beta = -1.3$ and $\phi = 0.01$

Dynamic predictions

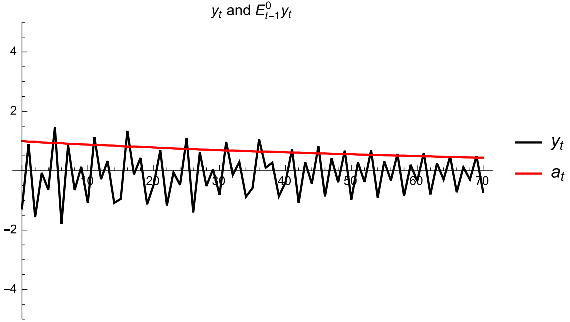


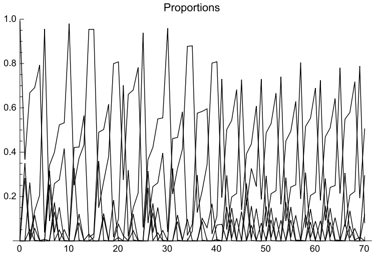


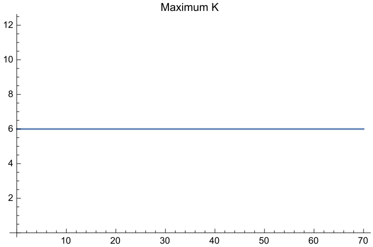


$\beta = -0.9$ and $\phi = 0.01$

Dynamic predictions





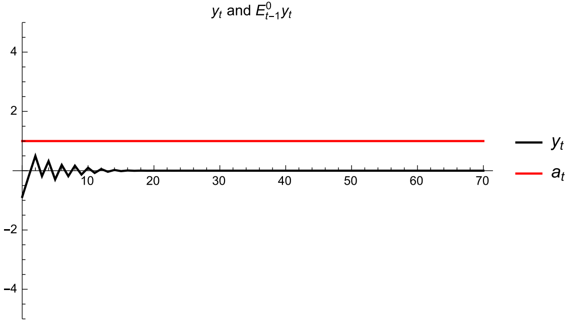


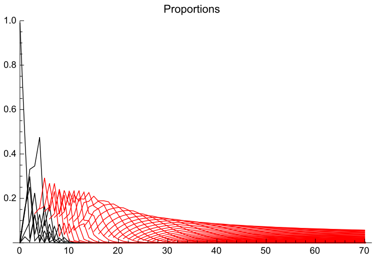
$\beta = -1.3$ and $\phi = 0.01$

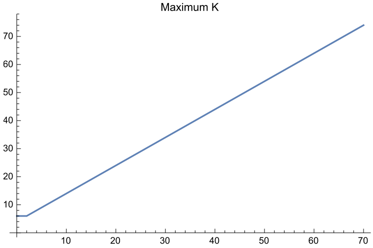
Dynamic predictions

We can also obtain convergence in the absence of any learning when $-1 < \beta < 1$ as discussed in theoretical results

Dynamic predictions







$\beta = -0.9$ and $\phi = 0$

Dynamic predictions in a forward-looking models

Announcements in a Forward-Looking Models

Level k beliefs imply a relationship between how forward-looking an agent *is* and how forward-looking he/she believes others *are*

$$
\begin{aligned}
E_t^k y_{t+1} &= \gamma_{t+1} + \beta E_{t+1}^{k-1} y_{t+2}, \\
E_{t+1}^{k-1} y_{t+2} &= \gamma_{t+2} + \beta E_{t+2}^{k-2} y_{t+3}, \\
&\vdots \\
E_{t+k-1}^{1} y_{t+k} &= \gamma_{t+k} + \beta a, \\
&\text{or} \\
E_t^k y_{t+1} &= \sum_{i=1}^{k} \beta^{i-1} \gamma_{t+i} + \beta^k a.
\end{aligned}
$$

Announcements in a Forward-Looking Models

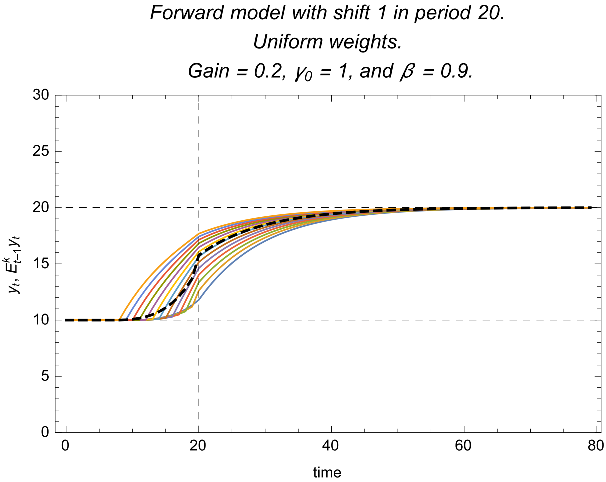


Figure: Forward-looking model with anticipated structural change at $t^* = 20$. Here $K = 11$, i.e. there are 12 types.

Concluding Remarks

Conclusion

- Propose a new model of bounded rationality
- Add a strategic and forward-looking component to an adaptive learning model
- Modeling assumptions are grounded in experimental research
- Model rationalizes behavior observed in the lab
- Model predicts muted heterogeneous responses to credible announcements about future structural change

Thank you!